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REVIEW OF OWN COMPLEX RESEARCHES RELATED TO BAR STRUCTURES

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ABSTRACT: In this year the author reach his age of 65. Therefore, it is proper moment to present whole his scientific achievements accumulated during the last 42 years of professional life and 39 years of scientific activity, oriented at all on bar structures. There, we can find investigations related to: analytical approaches of plane bar grids and double layer trusses: plane cylindrical and spherical; very general and effective theory for space bar structures, fundamental theory - common for single straight bars with any cross-sections, as well with thin-walled or full cross-sections (compact or solid, too); homogenous or composite. Moreover, the theory concern of global physical relations for whole bar (displacements of ends to internal forces) and stress calculations. The analysis can be led in the range of static, dynamics or dynamical-stability. At last, were proposed some analytical, numerical and hybrid effective approaches to solutions of very wide class of structures – practically with almost any: scheme, loading and support systems. Were considered motionless and moving loadings for bridges or airstrips, etc., too. In this algorithms were used originally elaborated Difference-Matrix Equation Method, new application of Finite Differences Method – 3D Time Space Method for dynamics and even application of standard commercial MS Excel tool. The theories were verified by certain own experimental researches.

Besides wide activity, concerning of mechanical behaviour of bar structures, was proposed by author some ideas concerning of morphology of bar structures oriented on large span roofs – plane and double-curvature - domes, including in it so called UNIDOM space bar system.

Summarizing above short enumeration of solved tasks, it seems, that it is worthy to give such review and present it in one publication.

Key words: Space bar structures, analysis, synthesis, experiments, computer programs, theories, thin-walled, composite structures, bars

1. INTRODUCTION

The paper presents short characteristics of scientific achievements and complete list of publications of the present author in 65 year of his life. The sequential chapters try to give more important information about the subject and domains of his activity, obtained results and to show importance of its application for civil engineering.

All scientific and professional activity are concentrated on providing new, better, complex theories for description, analysis and synthesis of as well single straight bars and first of all large, complicated space bar structures, too. All theories are oriented on computer methods and are applied in several dozen own prepared computer programs and systems.

The author was working several dozen years on designing of own computer systems with different destinations and its automation, all based on elements of own theories. They were produced mainly for scientific reasons Refs 30, 32, 35 etc. and are very effective, but never were of commercial destination. So, the paper presents experiences in domain of exploitation these computer program systems, destined for designing of various types of mainly large space bar structures. Most of these programs were oriented on large span roofing systems like: flat double layer bar structures, vaults and domes. They are enabling analysis and synthesis of many types space bar structures. Some of they provide automatic dimensioning of bars cross-sections, shape and form-finding of whole structures, with even optimisation elements. The bars of these structures can have any boundary condition. There are possible analyses: static, dynamic, stability and dynamical stability. Some other programs are oriented on experimental and theoretical analyses of particular bar with thin-walled or full- homogenous or composite bars. In these programs were applied mainly own, original theories from domains: strength of materials Ref 35, thin-walled bars Refs 16, 30, 32, composite bars Refs 16, 30, 32, 35, etc., space bar structures e.g. Refs 1-5, 8-21, 28, 29, 252 etc. or *3D-Time Space Method* (3D-TSM) for dynamics Refs 223, 236-238, 241 and numerical algorithms Refs 4, 5, 7,8,9, 18, 28, 29 etc.

Certain concise inspection and classification of whole activity of author are giving following keywords, grouped in some categories:

- analysis, synthesis, experiments, computer programs, theories, thin-walled, composite structures, bars, straight bars,
- uniform, theory, static, dynamics, stability, dynamical stability. strength of straight bars, moving loads,
- complicated, space bar structures, geometry, family, two-curvature, bar domes, regularity, wavy domes, free form,
- space bar system, UNIDOM, architectural forms,
- Computer Aided Design, numerical, comparisons, tests, evaluate, exactness, errors, safety.

2. EXPERIMENTAL ANALYSIS OF BAR STRUCTURES

Experiments permit to verify accuracy of taken assumptions for theories and obtained analytical or computer results. For the same reasons the author was starting his experimental researches.

The theoretical investigations of thin-walled bars and frames behaviour were supported by some experiments presented in literature, where to worthy of mention belong:

- experiments for single thin-walled bars with open cross-section, executed by V.Z.Vlasov Ref 311 (1940),
- for thin-walled rolled bars with open cross-sections, led by A.I.Strielbickaja, Refs 306-308 (1958, 1964, 1968),
- for straight cantilever beams with open or closed box type cross-sections, J.B.Obrębski Ref 30 (1991), Ref 35 (1997) Figs 1-9,
- for plane frames composed of 2 or 3 bars, N.Jankowska (Ph.D. thesis, supervised by J.B.Obrębski, Ref 282 (2006), Figs 14-16,
- for determination of critical bimoment value, (J.B.Obrębski with M.E. El.Awadi, Egypt, Refs 82, 194, 203, 277 (1992, 1993) Figs 10-13.

In authors some papers, the attention was focused on electro-resistance methods of measurements and on different approaches to elaboration of obtained results. Moreover, there are compared results of investigations presented in the book of A.I.Strielbitska and G.I. Jewsiejenko and the other own measurements done much later together with S.Wichniewicz, Z.Urbaniak, P.Flont, N.Jankowska and by A.Glinicka – all concerning of some thin-walled elements used by designing of civil engineering structures. There, are discussed applied methods of measurements, manner and form of elaborated results. Special attention was turned on advantages following of commercial program MS Excel application.

2.1. Behaviour of cantilever beams under combined loadings

The problem was discussed in some lectures presented by author in period of years 1988-2004. Lately author was coming back to wider elaboration of obtained very wide and numerous measured data.

The experiments, concern of cantilever beam behaviour under bending-torsion loading. The outlook and scheme of experiment are given in the Figs 1-6. As specimen were used of natural scale brazen thin-walled bars with approximate dimensions 10x20x196cm with walls thicknesses 0.5mm, 1.5mm and 2.5mm. There were measured strains in two cross-sections (in distance of 2 and 55cm from fastening), displacements and load capacity of the bar. Moreover, behaviour of the bar was observed and documented carefully. There were investigated 17 bars. From it, 13 with vertical position (as in Figs 3-6) and rectangular cross-section (together with Z. Urbaniak) and 4 with horizontal orientation (together with P. Flont, Ref 272). So, we can confirm, that before bar damage (breaking) were appearing some waves by fixed point (Figs 4c,6-8,10c) in both bar sides and in bottom wall. Some waves appear at loaded bar end, where longitudinal displacements were constrained by rigid cork, Figs 4C, 10c.

The electro-resistance measurements were executed by J.B. Obrębski and Z. Urbaniak in years 1988-96 (Refs 75, 76,79, 81, 82, 83, 85, 119, 183, 195, 197) on series of 10 thin-walled cantilever beams with rectangular cross-section mentioned above. Measurements were executed by means of two manually operated electro-resistance Wheatstone bridge devices with six distribution boxes for changing up to 132 electric channels for particular sensors. Zeroing of electro-resistance bridges for each sensor was done manually, rotating handwheel. The readings were manually taken down to tables, with columns for each loading level and rows for each sensor. In separate tables for longitudinal, cirquital and for oblique sensors.

Fig. 1. Author by his stand

Fig. 2. Stand with brazen cantilever model

The beams were loaded up to breaking. Dependently on thickness of bar walls were given loading levels: N=8-12 (brazen walls 0.5mm), N=13-20 (walls 1.5mm), N=14-18 (walls 2.5mm) Ref 183.

The other experiments were performed by P.Flont (supervisor J.B. Obrębski), on 4 similar cantilever beams with identical cross-section (thin-walled, open or closed). The longitudinal slit or closing were placed at top Ref. 272. Applied schemes of beams were identical as in previous, above experiments, Fig 4A. Results were elaborated manually.

Fig. 3. View on stand for bending-torsion loading of rectangular cantilever brazen beam. Four steel frames helpful by measurements of displacements are visible.

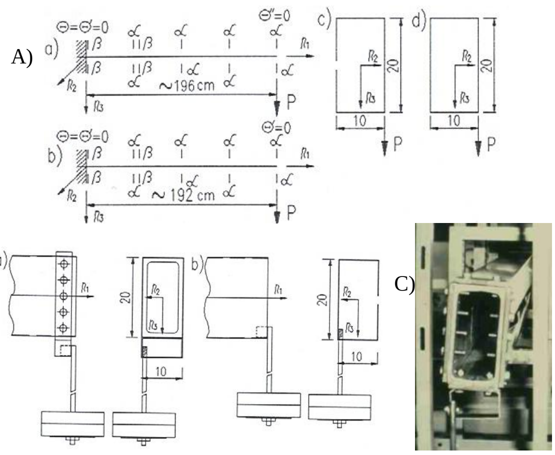


Fig. 4. Scheme of experiment of the Figs 2,3. A) Two applied schemes for cantilever beams: without constrains at free end a) and with planarly constrained displacements b), with rectangular cross-sections: open c) or closed d). B, C) Loading system for thin-walled bars.

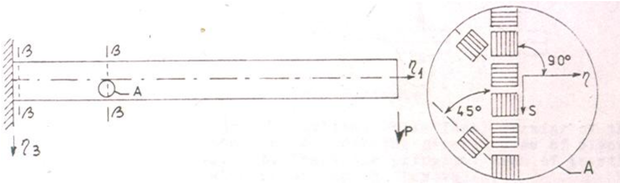


Fig. 5. Cantilever beam and electro-resistance (tissue paper) sensors arrangement (detail A)

During this type of experiments, were carefully observed developments of waves on side walls of bars, open and closed. Moreover, there were measured: deflections, rotations, strains, load capacity. Next, were compared calculated and measured values of bimoment.

Specially, interesting is photo of the Fig 8A, where are compared similar four bars, with identical thickness of walls (0.5mm). Two of them have open cross-section and bar opposite end (“free”- loaded), at all free or constrained by cork. Similarly, two next bars had have closed cross-section and bar opposite end free or constrained by cork, too.

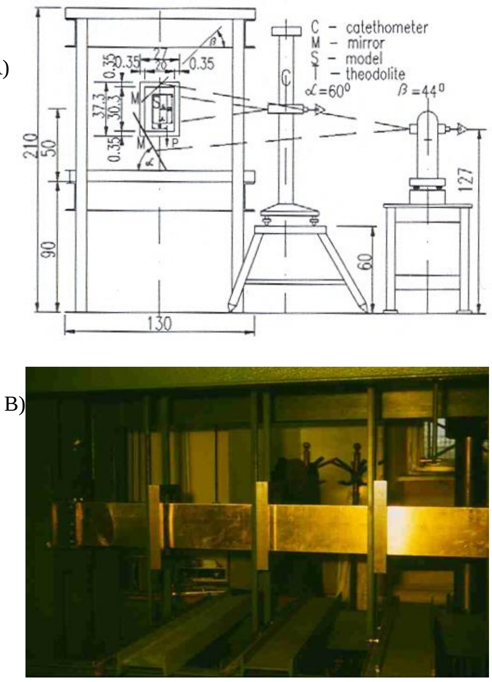


Fig. 6. A) Scheme of optical observation of circuital and longitudinal displacements; B) visible: cantilever beam and steel frames; at left – fastening and hinge (breaking point)

There is interesting **different distance of breakings from fixed end** (at down) and its inclination. Here we shall remember that for open type cross-sections, sectorial stresses are much higher. Simultaneously, disposition of longitudinal normal stresses for bars with open and closed cross-sections are dramatically other. Therefore, it results in other behaviour of the bars, shown in the Fig 8A. Additionally, the positions of hinge at lower edges of the bar can be explained by different diagrams of bimoments, too. Now, it is not in details discussed.

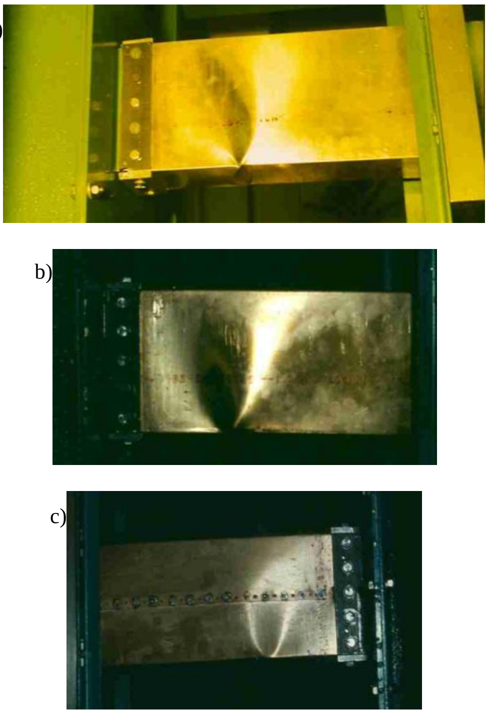


Fig. 7. Cantilever beam with planarly constrained fixed and "free" ends – zones of bimoment influence; a) waves by fastening on side and lower bar walls; b), c) hinge (breaking point) from both sides of bar

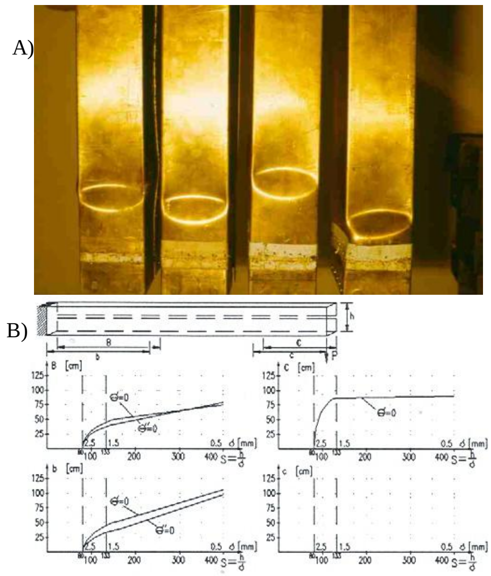


Fig. 8. A) Accordingly to schemes shown in the Fig 4A, the distance of lower part of hinge (breaking point) visible from down for all four cases of the bars with identical wall thickness (0.5mm); B) proper diagrams explaining positions of hinge at lower wall edges.

As consequence of above conclusions we obtain measured results of similar bars load capacity given in Table 1. There, bar with open cross-sectin obtain load capacity P_n=45.143kg, (with boundary condition at free bar end $\Theta'=0$) higher than bar wit closed cross-section P_n=43.883kg (with boundary condition at free bar end $\Theta''=0$). It is result of better boundary conditions of the first bar. Simultaneously, in the Fig 9, are given comparative curves showing character of dependences of bar load capacity on thickness δ of its walls. It is strongly nonlinear. We can draw conclusion, that **by thicker bar walls, influence of local instability is much smaller**.

Table 1. Measured critical loadings (load carrying capacity) for cantilevers with walls thickness 0.5mm, with open or closed cross-sections, with free- or planarly constrained right end

Scheme of the cantilever	P_n load capacity [kN] (hanging mass [kg]) by given bar cross-section δ=0.5 mm	
	(open section, 10 × 20, P)	(closed section, 10 × 20, P)
Θ=Θ'=0, Θ''=0, ~196 cm, P	302.858 (30.883)	430.345 (43.883)
Θ=Θ'=0, Θ'=0, ~192cm, P	442.702 (45.143)	488.793 (49.843)

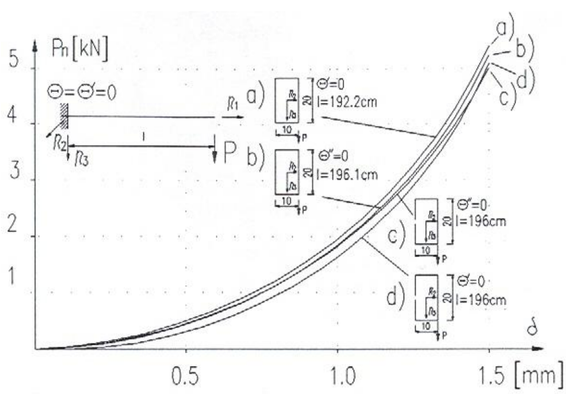


Fig. 9. Capacity of cantilever beams (see Table 1)

2.2. Measured torsional internal forces

It is worthy to explain, that after electro-resistance measurements of strains, were calculated normal and shearing stresses, which applied in Eqns (1) permits to determinate of measured bimoments B and bending-torsion moments, given in Table 2 (J.B. Obrębski, Ref 94).

The bimoment and bending-torsion moment have among the others, two following definitions:

$$B = \oint \sigma_1 \hat{\omega} d\overline{A} = \oint \sigma \hat{\omega} ds \,, \qquad M_{\hat{\omega}} = \oint \tau n \, ds \,, \qquad (1)$$

expressed by: stream of normal stresses σ or shearing stresses τ and its arm n. On basis of above formulae, J.B. Obrębski has calculated measured experimentally bimoments and bending-torsion moments presented in Table 2, Ref 94.

It is worthy to add, that shearing and normal stresses occurring in Eqns (1) can be experimentally measured, or calculated by any method, e.g. by FEM. This idea was used in Ph.D. dissertation of N.Jankowska, Ref.282.

Table 2. Internal forces - bimoments and bending-torsion moments calculated analytically and measured, accordingly to Eqns (1) (method proposed by J.B.Obrębski)

Internal Force	Model 21 with open cross-section		Model 22 with closed cross-section	
	analytical	measured	analytical	Measured
B [kNcm^2]	831.82	848.475	-4.826	9.65
M_ω [kNcm]	-10.1908	-2.175	2.189	3.35

2.3. Observed effects of bimoment

Bimoment belong to internal forces, unpopular by scientists and engineers. For the reason of difficult its explanation, in most of books and standards, is simply ignored. So it is important, to show experimentally its existence and effects. Such observations were undertaken by author.

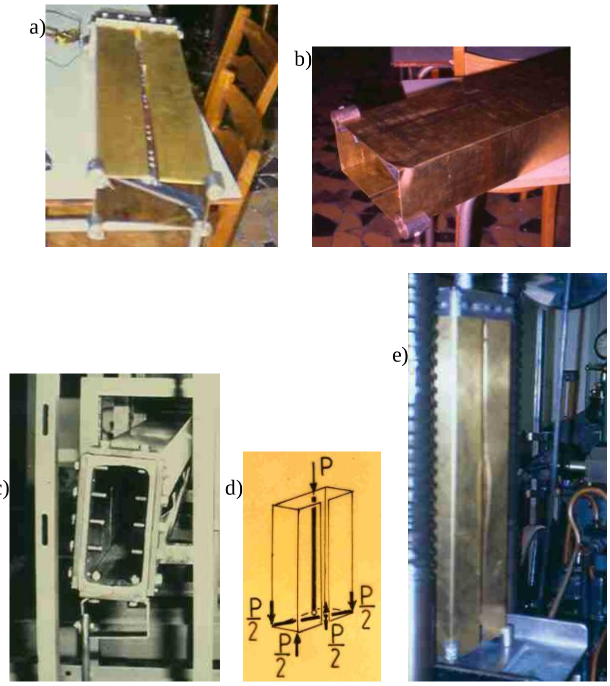


Fig. 10. a,b, d) loading system for bimoment, Visible waves on free longitudinal slits of: c) cantilever (visible strong steel cork); e) of bar loaded by pure bimoment at lower end (four equilibred forces).

In this chapter, are presented mainly the photos of the experiments performed in years 1988-1992 Refs 76, 79, 81, 82, 187, 195, 197 and elaborated partially a little latter. Some observations concerning observed phenomena were quoted in the books Refs 30, 35. Other materials were not published up to the moment. There, were performed two kinds of experiments, concerning cantilever beam and application of pure bimoment loading. So, especially in zones, where is acting bimoment, there were observed local instabilities of bars walls. Intensive values of bimoment, specially appears in thin-walled bars with open cross-sections, see Table 2. Therefore, **in bars with open cross-section and with thinner walls we observe more intensive waves**. These easy conclusions follow of the effects well visible in photos of the Figs 10c,e, 12, 13.

2.3.1. Bar loaded by pure bimoment

In second type of experiments, bar was loaded by pure bimoment, only. The experiment, concern of short rectangular bar, showed in the Figs 10d,e,11-13. Accordingly to scheme given in the Fig 10d, pure bimoment was applied, at down. At the top of specimen, in each case was applied rigid cork for constraining of longitudinal displacements and for stiffening whole model. Besides of it, there were applied three different type longitudinal slits: in the middle, in one quarter of wider wall and in the bar corner, Figs 12,13. Moreover, there were applied three cases of walls thicknesses: 0.5mm, 1.0mm, 1.5mm, see diagrams in the Fig 11.

At certain value of bar loading P (Fig 10d) appears waves along slot, Figs 10e,12,13. **Besides of waving of slots edges, in each case were observed certain rotations of the bars top end**. In these series of experiment, without any doubts, waves and critical loading were resulting of the pure bimoment application, only. So, it all confirms the thesis, that in the first type of experiments, bimoment is significantly responsible for wavy effects on side walls and along slits, too.

There, were measured critical forces – which brings longitudinal waves at longitudinal edges of open thin-walled bars. Next, critical bimoment B was calculated accordingly to equation (13) similar to Eqn (1a). Experimental results are compared in the diagram of the Fig 11, with similar ones calculated analytically by means of formulae derived by J.B.Obrębski in the book Ref 35.

For thinner bar walls, the convergence is enough good. For thicker bars, differences of results are much bigger. Experimental curves show high nonlinearity of the phenomenon. Contrary, the observed theoretical values of B_{cr} depends almost linearly on thickness of bar walls. In reality, similar values obtained experimentally are higher and evidently nonlinear.

So, we come to important conclusion, that **bimoment as force really exists and can be observed and calculated**. Moreover, in any case it is important, that the **bimoment has its critical value which can be measured and analytically calculated, with enough convergence!**

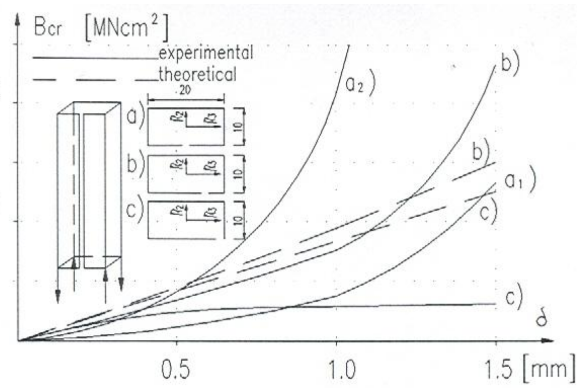


Fig. 11. Observation of critical bimoment (with M.E. El. Awadi, Egypt, Refs 194, 195, 82 (1991, 1992) for short, rectangular bar, with planarly constrained displacements by strong steel cork at top; a), b) loading system by pure bimoment at bottom bar end; c) small damage of supported corner; d) comparison of measured and calculated critical bimoment.

Fig. 12. The specimens, loaded in strength machine; visible three different type waves at free longitudinal edges of slit in middle of wider bar wall

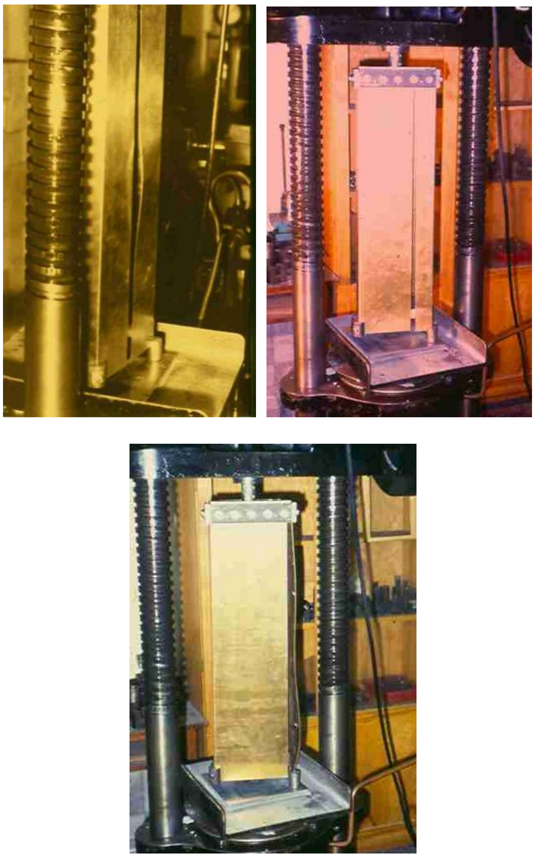


Fig. 13. Visible three different type waves at free longitudinal edges for bars with longitudinal slit on three positions of the wider wall

2.3.2. Experimental examples of bimoment influence on instability of thin-walled bars

All these experiments show high importance of torsion and bimoment in single bars mechanics, and for frames, too. Similarly, this chapter presents examples of visible bimoment influence on instability of thin-walled bars – local and global. There, own experiments concern of the bars loaded by pure bimoment or bended with torsion. Shown photographs and drawings presents observed effects. In the light of these experiments, the bimoment is evidently a real internal force, very dangerous for structures, which should be seriously considered by designing of objects composed of thin-walled bars.

2.4. Experiments on torsion thin-walled simple plane frames

The problem of computer analysis of space frames taking into consideration of bimoments, too, was numerically investigated by some authors. There, can be mentioned e.g. works by J.Rutecki Ref 303 (book 1957), J.H.Argyris and D.Radaj (1971), R.Dziewolski (IASS, Kielce 1973), J.B.Obrębski Refs 71,30 (1985, 1991), K.Grygierek (Ph.D. dissertation, Gliwice, 2003), and C.Szymczak et al. Refs 308, 309 (2003). There, still is serious question about real behaviour of space bar frames.

Therefore, N. Jankowska in she's Ph.D. dissertation Ref 282 (supervised by J.B.Obrębski), has investigated 8 models of simple thin-walled frames, type L, T and Y, Figs 14, 15, 16. The frames were composed of the 2 or 3 thin-walled bars, made of the brazen, connected with one centralnode, only. By both ends of each bar, in distance of 2.5 cm were glued 25 electro-resistance rosettes, Ref 282. Sensors in rosettes, were oriented along, transversely and under angle 45^o to bar axis. So, were noted by computer strains in these directions and next, calculated proper normal, shearing and principal stresses. There was applied very modern in that time electronic bridge of the firm VISHAY, system 5000.

All together, were investigated 32 cross-sections, in it most of them placed by central node.

A)

Fig. 14. Frame type L and loading system

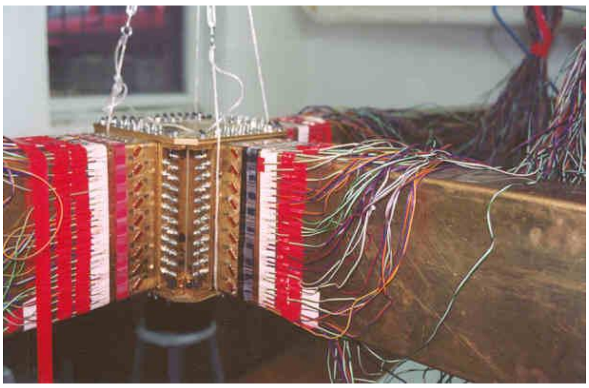


Figure 15. Electro-resistance measurements of T type thin-walled frame with bars having external dimensions 10x20cm.

Next, were drawn proper diagrams, and calculated among the other, internal forces, associated with torsion: B – mimoment and $M_{\hat{\omega}}$ - bending-torsion moment, according to formulae (1), by method proposed by J.B.Obrębski. Some results are given in Table 3, Ref 282. So, The transmition of bimoment through node and its dependence on node rigidity was confirmed.

B)

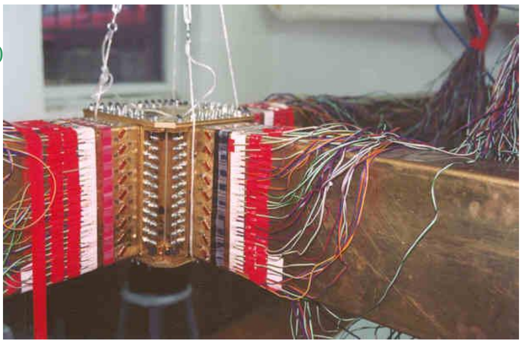


Figure 16. A) Electro-resistance measurements of L type thin-walled frame, with box bars having external dimensions 10x20cm; at the left is visible torsioning moment applied, only; B) electro-resistance measurements of T type of thin-walled frame

Table 3. Measured bimoments (N. Jankowska Ref 282, Table 4.17)

I – number of sheets in central node	Frame type Li			Frame type Ti			Frame Yi	
	1	2	3	1	2	3	1	2
	L1	L2	L3	T1	T2	T3	Y1	Y2
C –active CS	-1364	-1355	-1259	-1495	-1390	-1339	-1413	-1333
B-passive CS	-455			-285	-261	-241	-322	-285
E-passive CS				287	244	222	259	223

It is important, that similar observation concerning of bimoments transmission, obtained numerically by super-elements technique, was reported by C.Szymczak et al. (see Refs 309, 310).

2.5. Application of commercial program MS Excell to elaboration of experimental results

Lately, author come back to new complex elaboration of accessible own and quoted in Ref 282 measurements data. This time it is lead by means of commercial program *MS Excel*. For this purposes, were foreseen for each model separate document and for each cross-section individual calculation sheet. The shape of cross-section is there declared by coordinates (y,z) and all necessary material and model data. Therefore, such program can be applied for different types of cross-sections.

The MS Excell program is generating: all geometrical characteristics of cross-section, internal forces, strains, normal and shearing stresses, principal strains, angle of non-dilatational strain, principal stresses and its inclination. All these information will be presented during LSCE 2009.

Before calculation, the measured data were interpolated for points assumed in distance 1 cm each of the other. For points located on longitudinal free edges, measured data were extrapolated linearly, by assumption that shearing stresses τ_{xz} and normal – circuital stresses σ_z are equal zero, Refs 182, 183:

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_c) \;, \quad \sigma_z = \frac{E}{1-\nu^2}(\varepsilon_c + \nu\varepsilon_a) = 0 \;,$$

$$\tau_{xz} = G(2\varepsilon_b - \varepsilon_a - \varepsilon_c) = 0 \,, \tag{2}$$

or

$$\varepsilon_c = -\nu\varepsilon_a \;, \quad \varepsilon_b = \varepsilon_a \frac{1-\nu}{2} \;. \tag{3}$$

So, longitudinal strains ε_a were interpolated linearny, and remaining strains: cirquital - ε_c and inclined by 45^o - ε_b ,were calculated by means of Eqns (3). Example of some obtained results is shown in the Fig 17.

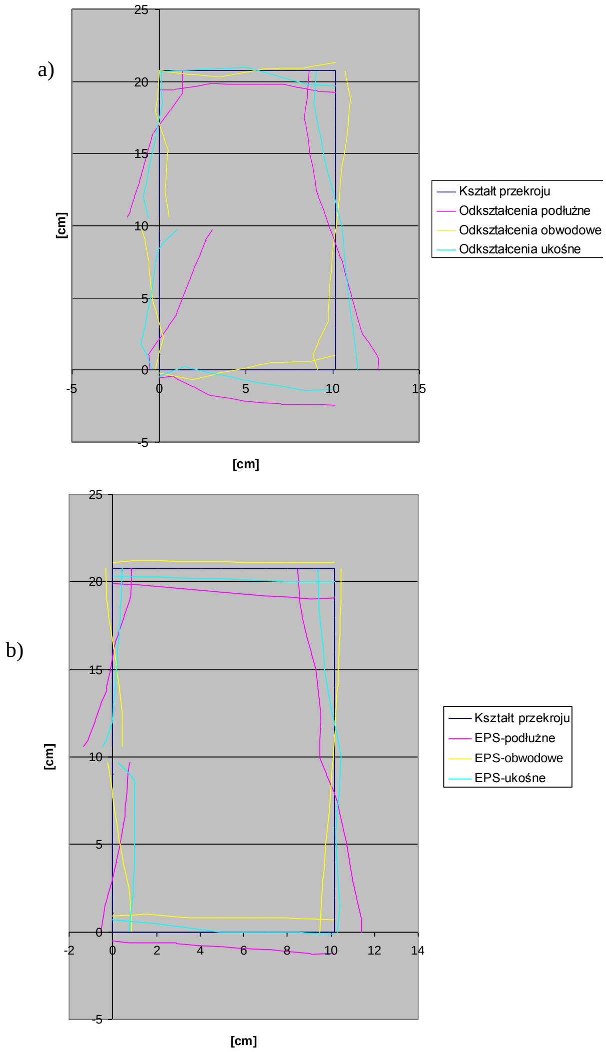


Fig. 17. Diagrams of measured strains for model No 11 with walls thickness app. 1.5 mm, loaded by hanging mass 100 kg; in cross-sections: a) 2 cm and b) 55cm from fastening black lines – shape of cross-section, red lines – longitudinal strains, yellow lines – cirquital strains, green lines – inclined strains.

2.6. Summary for experimental observations

After above discussion, on the ground of presented data and photographs, we can drawn following simple conclusions:

a. bimoment is identically real internal force as: longitudinal one, a shearing forces, and bending and torsion moments,
b. there can be calculated **critical internal forces:** longitudinal, bending moments (associated with transversal critical loadings) and bimoment (in consequence critical torsion moment) Refs 30, 35, 225, 227,
c. so, any type of bar loading can bring us to critical state of loading,
d. by designing of bars should be taken into consideration all types critical loadings, including combined loading, which can be easily calculated and considered (see Refs 30, 35).
e. In experiments described in chapters 2.1 and 2.3 dependently on particular thickness of walls were observed waves at other position, Figs 4, 7, 8A, 10c, 12, 13. Explanation of this problem is connected with bimoment activity, but now it is not wider discussed.

3. NEW THEORIES

Theories applied in all elaborated own programs, were built successively as it is given below. Each of them is shortly characterised as follow.

3.1. Theories for single straight bars

Considered bars can have any type of cross-sections (**CS**s): full or thin-walled (**TW)** of open or closed type, homogenous or composite (as e.g. in the Figs 18-20). Applying each of mentioned types of **CS**s, different will be determination of bar rigidity, including torsion properties, but global analysis of structures will be led by equilibrium equations and physical relations for particular bar, formally the same. It concern of all types of analyses: static, dynamics, stability and dynamical stability, all with influences of surrounding media, as in the books Refs 30,35.

It is suggested for **CS** composed of more than one material (composite) as e.g. reinforced shown in the Figs 18-20, to calculate so called **reduced** geometrical characteristics of the **CS**s, as it is given in the Table 4. So, there are not problems with determination of such characteristics associated with torsion: $\hat{\omega}_{\max}, \bar{I}_{\hat{\omega}}, K_s$ (see Refs 30, 35) and LSCE books). For example, deflections calculated for a beam with the **CS** of the Fig 20a are 94,9% bigger than for the identical beam, but with the **CS** of the Fig 20e (compare bold numbers in Table 4).

Geometrical characteristics of homogenous straight bar. In author's theories, generally the bars taken into consideration, can have the **CS**s homogenous or composed of some materials. There, material form strips or fibers disposed along the bar. The walls thickness around the **CS** circuit can be variable, Obrębski Refs 16, 30, 32 (1989, 1991, 1999). In the case of homogenous cross-sections, calculation of its area, center of gravity, moments of inertia follows in traditional way. The difference appears for composite-, perforated-, with lacings- and for multi-branched bars. Below are commented the first two cases.

Geometrical characteristics of composite bars. As it follows of the derivations of the theory, in the case of the CSs composed of some materials, should be assumed for whole CS (for whole structure) general Young's modulus $\bar{E}$ – the best as for the strongest material.

It results in introduction of reduced elementary area $\overline{dA} = (E_1/\bar{E})dA$ and next reduced geometrical characteristics of cross-section e.g.: area $\bar{A}$, moments of inertia $\bar{I}_2$, $\bar{I}_3$, warping (sectorial) moment of inertia $\bar{I}_{\hat{\omega}}$ etc.

In consequence, there is calculated e.g. reduced center of gravity – a little different of the traditional one. Certain example of calculated of geometrical characteristics for five CSs of the Fig 20 are shown in the Table 4. The first is homogenous, and next four have different number of reinforcing bars. So, the bending rigidity of the reduced moments of inertia, are growing. All the CSs are symmetrical, built of concrete and steel bars. The torsion rigidity K_s is calculated, too.

It is very interesting, that for compact – full **CS**s, can be calculated their geometrical characteristics, assuming, that whole bar is built of **TW** tubes, located one into the other. Accuracy of calculated this way characteristics are enough close to exact theoretical results (Obrębski Ref 97). There are not problems with determination of such characteristics associated with torsion as: $\hat{\omega}_{\max}, \bar{I}_{\hat{\omega}}, K_s$ (Obrębski Ref 30 (1991) and LSCE books Refs 31, 34-45(1995-2007)).

It is important, that by analysis of the bars with variable rigidity on their length, applying numerical solutions by means of Finite Differences, in particular **CS**s can be considered other characteristics.

Geometrical characteristics of perforated bars or with lacings. **In the case of perforated bars, with openings located along of the longitudinal strips, material separating openings, is replaced by hypothetical wall with new thickness and mechanical properties (material modulus), but with identical mass and deformations as for original part of the bar.**

Further, the analysis is led as in whole theory for composite bar, Refs 30,32 (Obrębski 1991, 1999).

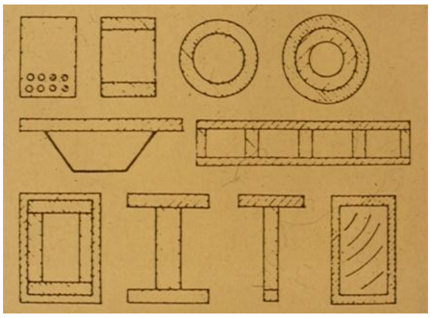


Fig. 18. Different type composite cross-sections

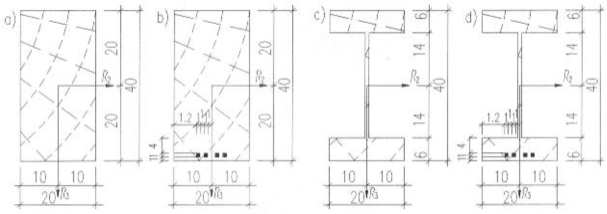


Fig.19. a, d) Homogenous and b, d) composite (reinforced timber) cross-sections

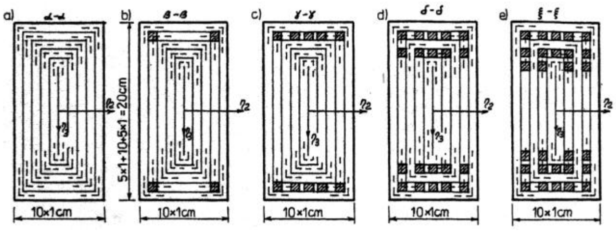


Fig. 20. Cross-sections divided on thin-walled tubes with walls thicknesses 1cm Refs 129, 236 (2000, 2002).

Table 4. Geometrical characteristics of the cross-sections of the Fig 20

Cross-section Of the Fig.	$\bar{A}$	$\bar{I}_2$	$\bar{I}_3$	$\bar{I}_{\hat{\omega}}$	K_s	$\hat{\omega}_{\max}$	$\hat{\omega}_{corner}$
	cm^2	cm^4	cm^4	cm^6	$kNcm^2$	cm^2	Cm^2
Figure 1a	200	**6666**	1666	8419	6062276	15.268	15.268
Figure 1b	218	8024	1898	12290	6190833	14.547	13.296
Figure 1c	246	10062	1942	20133	6422960	19.749	18.559
Figure 1d	293	12049	2219	26617	6663297	18.706	17.566
Figure 1e	331	**12991**	2494	28177	6935002	17.461	16.379

3.1.1. Uniform theory for thin-walled straight bars – possibilities and advantages

The theory was elaborated firstly about 1980 and published by the author in IJ *Thin-Walled Structures* Ref 16 (1989) and in the form of the book as lecture notes of Warsaw University of Technology Ref 30 (with second edition in 1999 Ref 32) under title *Thin-Walled Elastic Straight Bars.* Next it was extended in the book *Strength of Materials* (Ref 35), and in numerous conference papers. The theoretical derivations of the theory were supported by: many numerical calculations of comparative tasks, students homeworks, serious experimental investigations and by some problem-oriented own programs. The theory is lectured on the Faculty of Civil Engineering of Warsaw University of Technology since 1980.

The theory concern of the elastic thin-walled straight bars with any type of cross-sections: open, closed with one or more circumferences and open-closed. The **CS**s can be homogenous or composite – built of some different materials. There are derived uniform equilibrium equations for static, dynamics, stability and dynamical-stability, where are possible to be considered interactions with surrounding media as air, water or soil. So, range of structures analysis is here extremely wide and accuracy much better. The theory is completed by following particular problems:

- clear and easy algorithms for determination of geometrical characteristics of any thin-walled and full **CS**s, too (treated as set of thin-walled tubes located each in the other),
- simple analysis of stresses for composite bars (one formulae for normal stresses and one for shearing stresses),
- analysis of perforated bars or with lacings,
- calculation of critical combined loadings of any type, what brings the problem to ultimate critical curves or surfaces (instability of bending, torsion or bending-torsion character),
- there is possibility to calculate value of critical bimoment (Ref 35), too (what was checked experimentally),
- new proposals for calculation of stresses for bars with any type of **CS**, taking into consideration influence of its instability (see Eqns (24,25),
- theory of first and second order (including instability),
- theory of higher approximations – taking into consideration influence of shearing stresses disposition, on bar deformations, on internal forces and stresses,
- analysis of space frames by *DMEM* (see chapter 5.2.2), taking into consideration of bimoment existence (see Table 5 and chapter 3.3),
- it was derived exact stiffness matrix of *FEM* for **TW** bar Ref 30,
- dynamical behaviour of bars under moving loading applying of *3D-Time Space Method* combined with *Finite Differences* approach (applied to bridges, tall buildings, landing aircraft on airstrips, highways etc).

The theory was extended on the bars with full **CS**s (see above). So, independently on kind of **CS**s, the derived very general set of four differential equilibrium equations, always is the same. There are changed the values of calculated geometrical characteristics of bar **CS** and its mass, only.

The theory was tested by experiments (are excellent photos and unique results) and by *FEM* calculations, too. In each case were obtained sufficient analogies (any kind of analysis can not to give at all exact results).

Physical relations for straight bar. The internal forces, moment of free torsion M_s, bimoment B, and bending-torsion moment $M_{\hat{\omega}}$ depends on the first, second and third order derivatives of the function Θ describing of the bar longitudinal axial torsion angle (Eqns (4)):

$$M_s = K_s\Theta', \quad B = -\overline{EI}_{\hat{\omega}}\Theta'', \quad M_{\hat{\omega}} = -\overline{EI}_{\hat{\omega}}\Theta''', \tag{4}$$

$$M_1 = M_s + M_{\hat{\omega}} \ . \tag{5}$$

where: K_s and $\overline{EI}_{\hat{\omega}}$ are proper bar rigidity, (Obrębski Ref 30, 1991). So, the sum of free torsion moment and bending-torsion moment gives torsion moment - Eqn (5).

Besides above definitions of internal forces dependent on bar torsion angle Θ, in the theory were derived next expressions, (Obrębski Refs 30, 35 (1991, 1999)), given in Table 5, describing all internal forces for bar with number Λ $(1 < \Lambda < N)$ and with length l_Λ. There, are introduced bar rigidities defined as follow:

$$K_{0\Lambda} = \frac{\overline{EA}}{l_\Lambda}, \quad K_{1\Lambda} = K_s, \quad K_{2\Lambda} = \frac{\overline{EI}_2}{l_\Lambda}, \quad K_{3\Lambda} = \frac{\overline{EI}_3}{l_\Lambda}. \tag{6}$$

More information can be found in the book of J.B.Obrębski Ref 30 (1991, 1999). It should be completed by information, that in theory for space bar systems, were considered "finite dimensions of nodes" as in the Fig 21, too.

In formulae given un Table 5, coefficients $\overline{C}_i$ and $\overline{C}_i{}'$ are calculated from expressions dependent on bar boundary conditions at both bar ends and on kind of analysis: static, dynamics etc. Ref 11. Symbol $\vartheta_{1\Lambda} = \Theta'$, and Boole's operator E_Λ indicate the other end of the bar and its orientation in space (see Eqns 27, 28).

Table 5: Physical relations for internal forces for straight bar

	Internal forces in cross-sections by nodes
	Longitudinal and shearing forces
a)	$T_{1\Lambda} = K_{0\Lambda}(C_9E_\Lambda - C_{10})v_{1\Lambda}$
b)	$T_{2\Lambda} = \frac{K_{3\Lambda}}{l_\Lambda}\left[(\overline{C}_5{}'+\overline{C}_6{}'E_\Lambda)\varphi_{3\Lambda} + \frac{1}{l_\Lambda}(\overline{C}_7{}'E_\Lambda + \overline{C}_8{}')v_{2\Lambda}\right]$
c)	$T_{3\Lambda} = -\frac{K_{2\Lambda}}{l_\Lambda}\left[(\overline{C}_5 + \overline{C}_6E_\Lambda)\varphi_{3\Lambda} + \frac{1}{l_\Lambda}(\overline{C}_7E_\Lambda + \overline{C}_8)v_{3\Lambda}\right]$
	Torsional and bending moments and bimoment
d)	$M_{1\Lambda} = K_{1\Lambda}\left[(\overline{C}_{19}E_\Lambda - \overline{C}_{20})\varphi_{1\Lambda} - (\overline{C}_{21}E_\Lambda - \overline{C}_{22})\vartheta_{1\Lambda}\right]$
e)	$M_{2\Lambda} = -K_{2\Lambda}\left[(\overline{C}_1 + \overline{C}_2E_\Lambda)\varphi_{2\Lambda} - \frac{1}{l_\Lambda}(\overline{C}_3E_\Lambda + \overline{C}_4)v_{3\Lambda}\right]$
f)	$M_{3\Lambda} = -K_{3\Lambda}\left[(\overline{C}_1{}'+\overline{C}_2{}'E_\Lambda)\varphi_{3\Lambda} + \frac{1}{l_\Lambda}(\overline{C}_3{}'E_\Lambda + \overline{C}_4{}')v_{2\Lambda}\right]$
g)	$B_\Lambda = K_{1\Lambda}\left[(\overline{C}_{23} + \overline{C}_{24}E_\Lambda)\varphi_{1\Lambda} - (\overline{C}_{25}E_\Lambda + \overline{C}_{26})\vartheta_{1\Lambda}\right]$

On the basis of formulae describing internal forces, given in Table 5, in natural way was composed stiffness matrix of FEM for straight prismatic bar with practically any type of **CS**, Ref 30. The one only stiffness matrix is valid for different bars with any boundary conditions for all three functions describing deformed axis of the bar and for any kind of analysis. There, for particular bar and any applied range of analysis, the coefficients $\overline{C}_i$ and $\overline{C}_i{}'$ should be calculated in the other way, only, Ref 11.

The formulae in Table 5 were used by DMEM and for FEM (above) - for stiffness matrix of single bar, both elaborated by Obrębski, Refs 11, 30, (1991). Moreover, identical formulae can be applied for any bar with full or TW (open, closed, open-closed) **CS**s, Obrębski Ref 30 and LSCE 1995-2006.

In formulae d) and g) in the Table 5, we recognize other expression for the same torsion forces as in Eqns (1, 4, 5). It is now, yet once well visible, that **torsion moment d) and bimoment g) depends on the same displacements, and therefore, they appear always together!** This time, there are taken into account, bar boundary conditions. So, in the case of elastic behaviour of nodes, should be modified the coefficients $\overline{C}_i$ or $\overline{C}_i{}'$ to formulae of the Table 5.

Equations of motion. In discussed theory, were derived very general equations of motion taking into consideration theory of second order and interaction of the bar with surrounding media, Obrębski Refs 16, 30, 32 (1989, 1991, 1999).

The equilibrium equations for static or equations of motion for the theory of first order are the special - simplified cases of general equations. Only omitting proper terms, we create expected type of analysis, sometime truly unusual. Especially, it can be very advanced tasks, when we take into consideration interactions with surrounding media (three parametrical) – acting simultaneously, but independently in three directions.

Theory of second order can use whole – large set of equation or simplified its version, dependently on character of the considered task. Accordingly to type of loading, it can be calculated displacements for static or for forced vibrations in dynamics. It can permit to determinate critical configuration of loading (combined), too.

More on basic relations of theory for single straight bar. Above, were given essential definitions of theory for **tw** bars. Now, we observe, that internal forces, moment of free torsion m_s, bimoment b, and bending-torsion moment $M_{\hat{\omega}}$ are described by: first, second and third order derivatives of the function Θ - of the bar longitudinal axial torsion angle (Eqns (4)), (Ref 30, J.B.Obrębski 1991).

Sum of free torsion moment and bending-torsion moment gives torsion moment Eqn (5).

Similarly, formulae on longitudinal displacements, strains and stresses, are together dependent on Θ as in formulae:

$$\begin{aligned} u_1 &= v_1 - v_2'\eta_2 - v_3'\eta_3 - \Theta'\bar{\omega} \ , \\ \varepsilon_1 &= u_1' = v_1' - v_2''\eta_2 - v_3''\eta_3 - \Theta''\bar{\omega} \ , \\ \sigma_1 &= E\varepsilon_1 = E\left[v_1' - v_2''\eta_2 - v_3''\eta_3 - \Theta''\bar{\omega}\right] \ , \end{aligned} \quad (7)$$

where new symbols means: v_i and Θ – means displacements and longitudinal rotation of bar axis (around shearing centre), η_2, η_3- coordinates. All above quantities depend on bar torsion Θ.

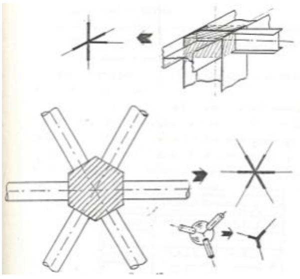


Fig. 21. Nodes with real – „finie dimensions" (see Ref 11 and Fig 55)

Analysis of simple structures built of thin-walled bars. Determination of internal forces for set of thin-walled bars or even critical combined loading belong to rather difficult tasks. For single straight bars it is possible to obtain exact solutions. Much difficult is matter of analysis for frames and especially for space bar structures.

The analytical solutions concern rather very simple tasks with simple system of loading and easy boundary conditions. There are some mathematical problems with solution of some sets of differential equations. Moreover, for the long bars, especially with closed **CS**s are problems with hyperbolic functions sinh x and cosh x (x>224). Actually, in comparison with numerical analyses based on FEM, FDM and DMEM, such solutions are rather not competitive. As profitable side of analytical method is existence of some closed formulae for determination of internal forces in the bar. Contrary, as weak side of such solutions, is to simple information about stresses and displacements distribution.

A) B) C) D)

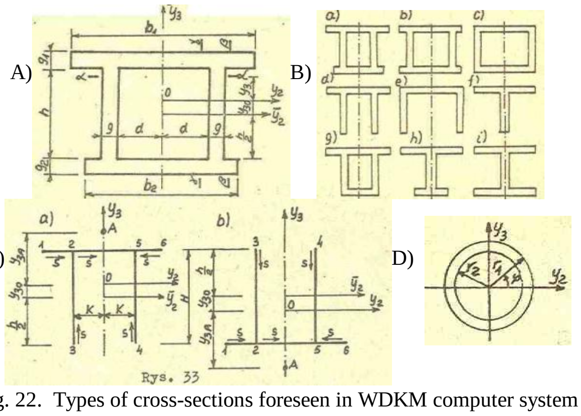


Fig. 22. Types of cross-sections foreseen in WDKM computer system for automatic dimensioning of bars (see Fig 70, too), Ref 11.

3.1.2. Theory for solid straight bars

In the book Ref 35 (1997) the uniform theory for straight bars with any cross-section was given. It is worthy to point **CS**s possible to be used in WDKM program system, Fig 22, too. Additionally it was investigated, extended and verified in many next papers. The theory has identical as for thin-walled bars (Ref 30, 32):

- equations of motion (equilibrium equations),
- formulae for displacements of the bar,
- formulae for stresses calculation (if no torsion – traditional),
- all solutions for static, dynamics, stability and dynamical stability.

The only differences concern of calculation:

- geometrical characteristics of cross-sections,
- rigidities of the bar including interactions with surrounding media.

3.1.3. Application of Finite Differences to analysis of complicated tasks

It was truth discovery, that this method can be still young, very effective and still competitive in certain applications, to FEM. It permit to analyze very complicated and real tasks with arbitrary boundary conditions, with variable rigidity on the bar length and bars under action of much more complicated external loadings.

Thanks of proposed application of ***F****inite* ***D****ifferences* ***M****ethod* for solving equilibrium or motion equations or their sets, are open possibilities to consider: very complicated systems of bars (Figs 22, 70), boundary conditions, combined loadings, very long bars, any type of interaction of bar with surrounding media etc. There is no problem with well known limitation of argument for hyperbolic functions sinh(x) and cosh(x). This way, the extremely advanced theory can be in whole range applied for plenty of complicated tasks, such, which were not possible to be solved previously.

So. the method can be applied to determination of the internal forces, displacements, critical combined loadings, etc. Moreover, the FDM uses the stiffness matrix of the structure in many cases much smaller than by ***F****inite* ***E****lement* ***M****ethod*. This way, solutions can be obtained even by *MS Excel* for some enouhgly big tasks, where besides of essential calculations, proper diagrams can be drawn.

Applying FDM, any task, which has theoretical solution in the form of differential equations, can be transformed to Finite Differences Operators (**FDO**), ever in the polynomial shape:

$$C_r\left(A_{ro} + \sum_{\Lambda=1}^{n} A_{r\Lambda}E_\Lambda\right)\Phi_r = Q_r \quad , \quad (8)$$

where, the symbols A and C means proper coefficients, e.g. Refs 155, 210, 223, 232, 249.

In result we come to solution of Eqn (9):

$$Kx = Q \ , \quad (9)$$

where unknown displacements x are determined, by given set of nodal forces Q and K – stiffness matrix of whole structure.

We can risk of thesis, that this method can give the same and even better results than theoretical ones and even as ***F****inite* ***E****lements* ***M****ethod*, too. There is possible to analyse the structures using:

- the highest quality differential equations transformed to FDO,
- easy in application program MRS (author J.B.Obrębski),
  there for one „node" can be used sets of equilibrium equations;
  for example for straight bars four equations.

This way, by this method we can start from differential exact physical relations and to have the same number of unknown displacements on "node" as when applying FEM.

3.2. Theory for plane hexagonal structures

Some interesting tasks were solved in author's Ph.D. dissertation, Refs 1-5 (1971, 1972). There, were derived equilibrium equations and proper elements of geometry for following structures:

- circular plane hexagonal grid, Figs 26-31, Refs 1, 2, 4,
- double-layer plane space bar truss type I (Fig 32, Refs 1, 2, 4),
- double-layer plane space bar truss type II (Fig 33, Refs 3, 4),
- cylindrical double-layer space bar truss type II (Fig 34, Refs 3, 4),

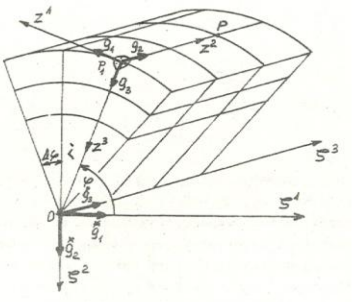


Fig. 23. Definition of cylindrical net of points, Ref 4.

There, were derived equilibrium equations of repeatable nodes and proper elements of geometry, e.g. Figs 23-25. Moreother, there was defined special mathematical calculus for solving analytically sets of finite differences equations with functional $a_{\Lambda i}$ - exponents of Boolea's operators (compare Eqns 27, 28)).

In result, for plane hexagonal grids, were found closed expressions defining internal forces and deflections, shown in the Figs 26-31.

Similarly, for double layer trusses, solution and analysis of numerical efficiency of some approaches, were done in Refs 4, 5, for some structures shown in the Figs 32-34.

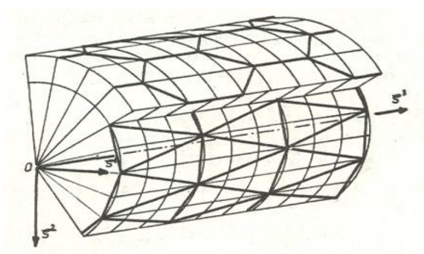


Fig. 24. Example of double layer structure inscribed to cylindrical net.

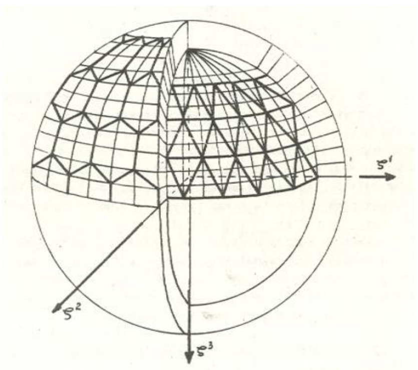


Fig. 25. Example of double layer structure inscribed to spherical net.

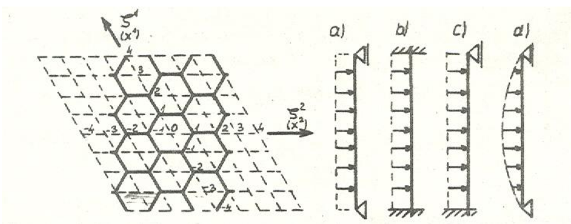


Fig. 26. Hexagonal band plate grid - considered schemes and loadings.

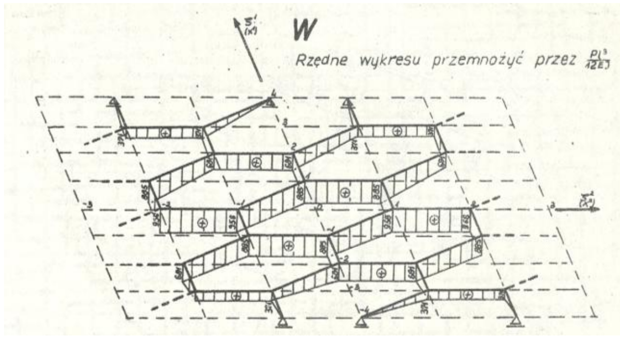


Fig. 27.Deflections of hexagonal band grid should be multiplied by $\frac{Pl^3}{12EI}$

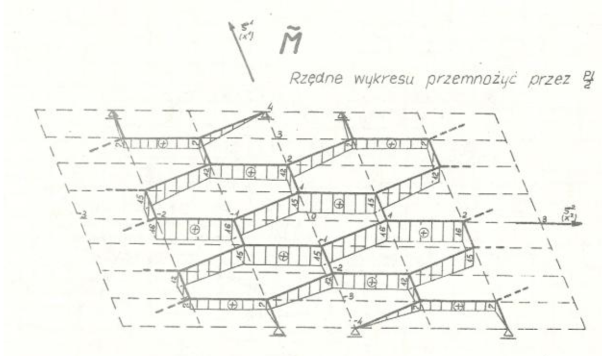


Fig. 28. Bending moments of hexagonal band grid should be multiplied by $\frac{Pl}{2}$

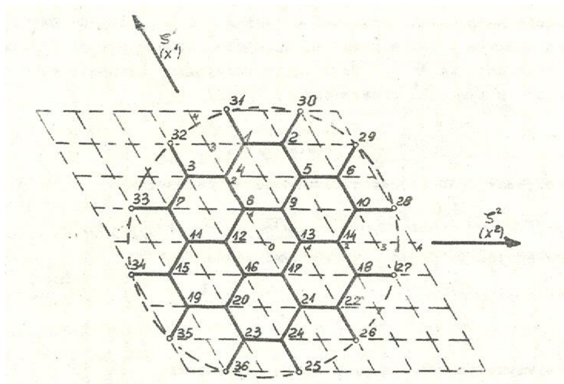


Fig. 29. Circular hexagonal plane grid in inclined net of points.

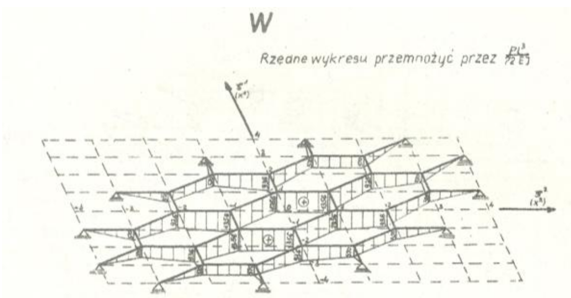


Fig. 30. Deflections for circular grid should be multiplied by $\frac{Pl^3}{72EI}$

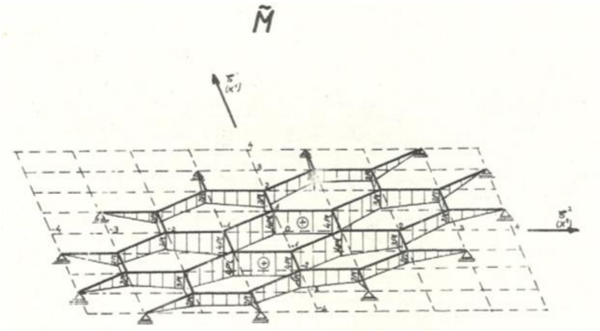


Fig. 31. Bending moments for circular grid.

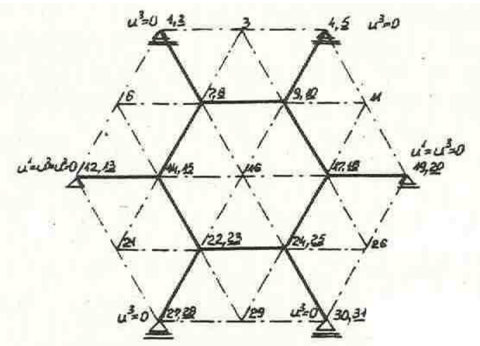


Fig. 32. Double layer hexagonal truss type I; continuous lines means bars in layers, dash & doted lines means bracings. Underlined numbers are refered to lower nodes.

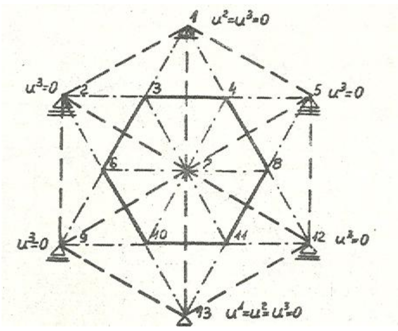


Fig. 33. Double layer hexagonal truss type II; continuous lines means bars in upper layer, dashed – lower layer, dash & doted lines means bracings.

3.3. Theory for any space bar structures

On analysis of space frames. Besides of discussion of single bars and elementary frames mechanics, highly important and extremely difficult, is problems connected with analysis of large space bar frames. Therefore, next theories oriented on analysis of such structures should be mentioned.

There, we can recognised certain tendencies to consider equilibrium of bimoments acting on nodes as sum of: algebraic, scalars, or vector character; with different interpretation.

They were elaborated by: J.Rutecki, certain general proposal, only, Ref 303 (1957); FEM application by J.H.Argyris and D.Radaj 1971; R.Dziewolski -for large roofs space bar structures (IASS, Kielce, Poland1973); J.B.Obrębski -for any large, complicated space bar systems, including frames Refs 71, 30 (1985, 91); F.Romanów 1988; K.Magnucki and W.Szyc, 1997; C.Szymczak et al. (application of FEM combined with super-elements) Refs 308, 309 (2003), K.Grygierek 2003 (Ph.D. thesis - FEM with seven degrees of freedom in node), etc.

To the approaches, in which bimoment was treated as vector, often in different ways, belong elaborated by: J.Rutecki, J.H.Argyris and D.Radaj, R.Dziewolski, J.B.Obrębski, C.Szymczak.

For dynamics of bridges under moving loadings, is recommended *3D-Time Space Method* by Finite Differences Method, J.B.Obrębski & R.Szmit Ref 223 (2000), J.B.Obrębski e.g. Refs 232, 237, 238, 240, 241 (2002-2004). Its application is extremely efficient, but not to the end exact (dependent mainly on applied space division). The methods belong to displacement method, too.

The author’s scientific investigations were led from 1969 till now, step by step in sequence, from space bar structures Refs 1-16, through thin-walled bars and bars with any cross-section Ref 30, over strength of any straight bars with any cross-section Ref 35, to stability and dynamical stability of various types bars, plates etc. – specially with moving loading by any manner (history) of its disposition in 3D space and in time.

This theory is elaborated for large space bar structures. It can be applied to discussed in chapter 3.2 plane hexagonal grids, or to any complicated space bar systems, but applying computers, e.g. Refs 11, 28, 29, Figs 45-52.

The paper, for reason of it’s to big volume, is limited to main topics mentioned above.

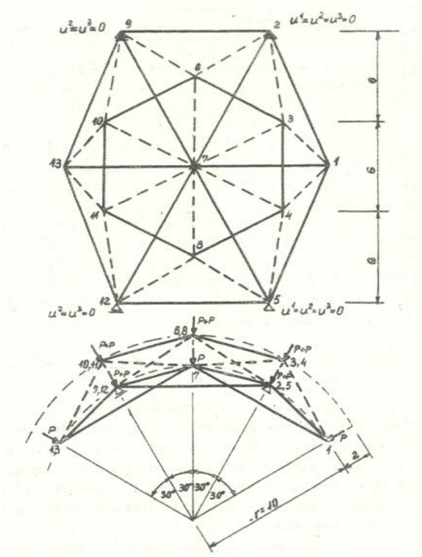


Fig. 34. Double layer hexagonal truss type II; continuous lines means bars in layers, dashed – lines means bracings.

Vector interpretation of internal forces. It is worthy to turn the attention on proposal for any large, complicated space bar systems, including frames, J.B.Obrębski Refs 71, 30, 32 (1985, 1991). The approach is based on vector interpretation of all internal – cross-sectional forces including bimoment and bending-torsion moment, follows in natural way from theory of **TW** bars, where B and M_ω are collinear with longitudinal bar axis of shearing centres of **CS**s. There, equilibrium equations of node are derived on basis of vectorial equilibrium of forces. It brings task to approach based on FEM or DMEM (***D****ifference*-***M****atrix* ***E****quation* ***M****ethod* – elaborated by Obrębski), both destined for structures composed of many bars with any type **CS**s and with any boundary conditions for each of four displacement functions. It concern of bars under any type of combined loading, in range of static, dynamics, theory of first and second order, too. The method seems to be the most exact for space frames, where bimoments as internal forces are calculated, too (Refs 30, 32).

More on vectorial equations. It is now evident, that all formulae of theory for bar structures can be considered as certain vectorial expressions. For example, previous formulae (4, 5, 7, 15) can be written in vectorial interpretation, e.g.:

$$\overline{EI}_{\hat{\omega}}\vec{\Theta}^{IV} - K_s\vec{\Theta}'' = \vec{m}_1 + \vec{b}' ,$$

$$\vec{M}_s = K_s\vec{\Theta}' ; \quad \vec{B} = -\overline{EI}_{\hat{\omega}}\vec{\Theta}'' ; \quad \vec{M}_{\hat{\omega}} = -\overline{EI}_{\hat{\omega}}\vec{\Theta}''' ; \quad \vec{M}_1 = \vec{M}_s + \vec{M}_{\hat{\omega}} ,$$

$$\vec{B} = \oint \vec{\sigma}_1 \hat{\omega}\, d\vec{A} = \oint \vec{\sigma}\hat{\omega} ds , \qquad \vec{M}_{\hat{\omega}} = \oint \vec{\tau} \times \vec{n}\, ds ,$$

$$\vec{u}_1 = \vec{v}_1 - \vec{v}_2' \times \vec{\eta}_2 - \vec{v}_3' \times \vec{\eta}_3 - \vec{\Theta}'\hat{\omega} ,$$

$$\vec{\varepsilon}_1 = \vec{u}_1' = \vec{v}_1' - \vec{v}_2'' \times \vec{\eta}_2 - \vec{v}_3'' \times \vec{\eta}_3 - \vec{\Theta}''\hat{\omega} ,$$

$$\vec{\sigma}_1 = E\vec{\varepsilon}_1 = E\left[\vec{v}_1' - \vec{v}_2'' \times \vec{\eta}_2 - \vec{v}_3'' \times \vec{\eta}_3 - \vec{\Theta}''\hat{\omega}\right] .$$

This matter as evident, therefore will be here not continued.

Analysis of space frames. As it is well known, the problem is extremely difficult and probably exact solutions by application of bars model to 3D type tasks, is impossible. However presented below approach, based on idea of vector sum of all forces acting on node, seems to be actually the best.

Some vectorial equations. Obrębski in Refs 71, 28, 29, 30, 178 (1985-2007) has shown derivations of theory, which indicates possibility to interpret all internal cross-sectional forces as vectors, including bimoment and bending-torsion moment, too. In consequence it was given proposal to consider equilibrium of forces acting on node as vector sums of proper forces, moments and bimoments. This assumption brings us to three following vectorial equations (Rutecki, Ref 303 (1957), Obrębski Ref 71 (1985)):

$$\sum_{\Lambda=1}^{N}\sum_{i=1}^{3}\vec{T}_{\Lambda i} + \vec{F} = 0 , \quad \sum_{\Lambda=1}^{N}\sum_{i=1}^{3}\vec{M}_{\Lambda i} + \vec{M} = 0 , \quad \sum_{\Lambda=1}^{N}\sum_{i=1}^{3}\vec{B}_{\Lambda i} + \vec{B} = 0 . \qquad (10)$$

Each of above internal force can be expressed by basis vectors $\vec{t}_{\Lambda i}$ collinear with bar longitudinal axis and with both principal axes of its **CS**, together with the values of internal forces calculated by means of formulae of the Table 5, Refs 11, 28, 29, 30:

$$\vec{T}_{\Lambda i} = \vec{t}_{\Lambda i} T_i , \quad \vec{M}_{\Lambda i} = \vec{t}_{\Lambda i} M_i , \quad \vec{B}_{\Lambda i} = \vec{t}_{\Lambda i} B_i . \qquad (11)$$

After projection of above three vectorial equations on three directions (r=1,2,3) of local (nodal) reference coordinates, we obtain nine equilibrium equations of node:

$$\sum_{\Lambda=1}^{N}\sum_{i=1}^{3} t_{\Lambda ir} T_{i\Lambda} + F_r = 0 , \quad \sum_{\Lambda=1}^{N}\sum_{i=1}^{3} t_{\Lambda ir} M_{i\Lambda} + M_r = 0 , \quad \sum_{\Lambda=1}^{N}\sum_{i=1}^{3} t_{\Lambda ir} B_{i\Lambda} + M_r = 0 . \qquad (12)$$

Using here expressions of Table 5 (physical bar relations displacement→force), and after some derivations, we come to DMEM. There, are nine equilibrium equations with nine degrees of freedom. For the reason of strong dependences between bimoment (Table 5g) and torsion moment (Table 5d), equations $(9)_2$ and $(9)_3$ should be used always together. So, $(9)_3$ can be regarded as additional conditions to equilibrium of moments $(19)_2$. In practical calculations the equations $(10)_2$ are dominating.

In all author's theories and programs, were used equilibrium equations derived from condition, that vector sum of all forces acting on node is equal zero (see Eqns (7-9). In result in computer programs, as central point of algorithm is foreseen solution of linear algebraic set of equations, written in the form of well known Eqn (9).

Extension of this approach, step by step gives works Refs 11, 184-186, 190 etc. There, bars of the space structures can be pin joined (truss) or fully rigid (frame) or with practically any boundary conditions at the nodes. Analysis can concern static, dynamics and in certain range stability of structural systems. On the basis of this theory were gradually elaborated programs *KM, KMT$_D$, KMT$_G$, WDKM* (Ref 11) and *SPES* (Refs 217, 233, 279). There, was used original method of stiffness matrix composition called as ***D****ifference-****M****atrix* ***E****quation* ***M****ethod* (DMEM). The square matrix K for Eqn (9) is built always by means of matrix type equation describing equilibrium of whole node of considered structure:

$$\sum_{\Lambda=1}^{N}(W_{\Lambda}^{o} + W_{\Lambda} E_{\Lambda})x = q \qquad (13)$$

Symbols W_{Λ}^{o} and W_{Λ} are matrices with dimensions 2x2, 3x3 or 6x6, dependently on task type. When scheme of structure is more complicated - of mixed type, dimensions of these matrices can be 2x3, 3x6, 6x3 etc. (Ref 28, 29). Here, in nodes are assumed identical numbers of degrees of freedom as in FEM. Often physical relations for particular bars are here more precise and for all kinds and ranges of analyses are written in the same manner Refs 136, 146. It permits to obtain almost exact numerical solutions.

3.4. Torsion in analysis of bar structures

The problem concerning of analysis of space bar structures is extremely interesting, difficult and of the highest scientific importance. The chapter presents results of the theoretical, numerical and experimental investigations, of previous authors and own, too. It is oriented rather on foundations of theory, on numerical calculations and efficiency of applied methods. It gives certain historical review of theories in domain and tries to show state-of-the-art of research works in the field of computer analysis of space frames composed of thin-walled or with full cross-sections straight composite bars. It recommends vectorial interpretation of bimoments for analysis of space bar structures.

Below is presented certain resume of many observation following of experiences concerning specially a torsion effects in analysis of bar structures. There are considered five essential topics:
- properties (including torsion rigidity) of bars used in structures;
- experimental observation of torsion influence on structures behaviour;
- analytical determination of critical bars loading;
- torsion in analysis of space bar structures;
- and at last – part of torsion by stresses calculation.

It is shown, that on all above designing steps, applying nowadays approaches can be generated remarkable or even terrible errors. Contrary, by application of proposed approaches, they can be essentially reduced. For above reasons, there is recommended application of some theories developed by author, concerning of:
- bars torsion, including homogenous-, composite and thin-walled (**TW**) with open or with closed cross-sections (**CS**);
- vectorial approach to composition of equilibrium equation of nodes for space bar structures;
- application of uniform criterion for bar instability even for combined bars loading;
- and at last, common approach to stresses calculation for any type bars – even with composite **CS**s.

More essential formulae. The main subject of this paper is to show some observations concerning effects generated by internal force called as bimoment B. It is defined accordingly to theory of **TW** bars in three ways Refs 30, 35: by means of Eqns $(1)_1$ $(4)_2$ and as below:

$$B = \sum_i P_i \hat{\omega}_i , \qquad (14)$$

where: $\hat{\omega}$ - generalized sectorial coordinate of force P_i position. Simultaneously, bending-torsion moment as a main part of bar torsion moment is defined by Eqn $(4)_3$.

So, from definitions Eqns (4) connected by Θ, follow next observations:
- **when torsion exist, in bar are observed torsion moment and bimoment, together**,
- when is applied bimoment, should be observed bar torsion and as internal force can occur torsion moment (when torsion is constrained),
- when torsion moment is applied, should be observed bar torsion and as internal force can occur bimoment (when longitudinal displacements are constrained).

Torsion in analysis of space bar frames –review and discussion. It is worthy to show certain historical review of theories in this domain and to give short state-of-the-art of research works in the field. It concern of computer analysis of space bar frames composed of **TW** or with full **CS**s straight composite bars. It recommends vectorial interpretation of bimoments.

The torsion of single bars and space frames are discussed. There, can be mentioned first theoretical solutions of de Saint-Venant for free torsion of prismatic bar, Prandtl’s membrane analogy or solutions by S.P.Timoshenko for bars with rectangular or triangular **CS**.

Next, it should be mentioned theory for thin-walled (**TW**) homogenous straight bars by A.A.Umansky 1939; with open CS by V.Z.Vlasov Ref 310 (1940), K. Roik, Carl and J. Lindner 1972, W. Gutkowski 1973, C.F.Kollbrunner and N. Hajdin 1975, T. Lewiński 2005 etc.

The Vlasov’s theory Ref 310 was extended by J.B.Obrębski Refs 30, 35 (1991, 1997) on composite bars with any **CS**. It provides own original theory for bars with TW or full **CS**s, homogenous or composite, presented in two books and lectured on Warsaw University of Technology. It is discussed below mainly. It can concern of any straight bars with any **CS**s: polygonal tubes, box girders with one or a few circuits, rolled and coldly formed bars, etc. (e.g. Figs 22, 70).

Own proposal makes possible to apply theory for any bar structures made of each kind of material as wood, steel, aluminium, glass, different composites, reinforced concrete etc.

Next, in literature it can be found different approaches to analysis of bar frames. In most cases it concern of simple plane frames. The main topic of the next chapters’ concern analysis of space frames taking torsion into consideration.

Torsional forces for single straight bar. With torsion are associated such internal forces as bimoment and bending-torsion moment, responsible for: warping of the particular **CS**s; for global warping of the whole bar by bending or by instability and at last on warping stresses – which can obtain very significant values. Torsion has strong influence on values of critical loadings, by combined loading, too. By combined loading of the bar appear all internal forces: bending moments, longitudinal and shearing ones and bimoments.

Our discussions should be started from differential equilibrium equation (15), one of the four (given here in the simplest form, Ref 30), responsible for bar torsion. There, function Θ describing of longitudinal axial torsion angle is dependent on external loading – continuous torsioning moment m_1 on external bimoment b and on bar boundary conditions:

$$\overline{E}\overline{I}_{\hat{\omega}}\Theta^{IV} - K_s\Theta'' = m_1 + b' \cdot \qquad (15)$$

In theory of the second order it is well visible, that set of four equilibrium equations of single bar (extended version of Eqn (15) dependents on longitudinal forces on transversal loadings and on bending moments, too (Ref 30, J.B.Obrębski, 1991).

Numerical example of torsion for straight bar. The example concern of straight **TW** bar with the length l=400cm and any **CS**. It is made of steel with properties of material E=205GPa, (E_1=225,27 GPa), G=80GPa, $\nu = 0{,}3$. At both ends it has torsion freely constrained ($\Theta = \Theta'' = 0$), Fig 35a. The task was solved analytically and numerically – applying DMEM (see chapter 5.2.2 and Ref 30).

By analytical solution it were used eight boundary conditions (2×4) for two bar sections AC and CB. On each of section were used other torsion function: $\Theta(\eta)$ and $\overline{\Theta}(\eta)$. The used boundary conditions are shown in Table 6, (a=b=0.5*l*). So, at central node, Fig 35b, condition of identical warping normal stresses (Table 6, position 3) bring us to equality of bimoments (Mutermilch & Kociołek 1964, and Ref 30):

$$B(a) = \overline{B}(a) \quad , \qquad (16)$$

or to equality of second order derivatives for torsion angles
$\Theta''(a) = \overline{\Theta}''(a)$ - see Table 6.

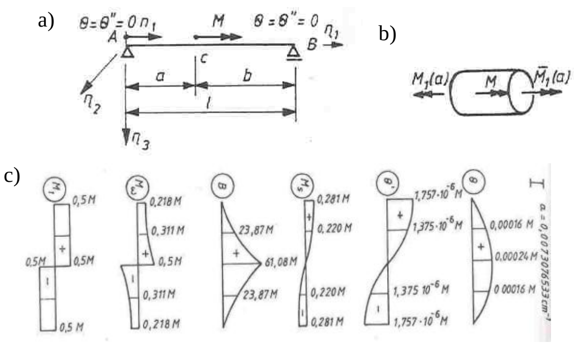


Fig. 35. a)Bar torsioned by concentrated moment; b) central node; c) calculated internal forces.

After calculations, we obtain results as in the Figure 35c. **There is well visible equilibrium of bimoment on whole length of the bar.**

Table 6. Boundary conditions

	Section AC $\eta = 0$	Node C $\eta = a = 0.5l$	Section CB $\eta = l$
1	$\Theta(0) = 0$	$\Theta(a) = \overline{\Theta}(a)$	$\overline{\Theta}(l) = 0$
2	$\Theta''(0) = 0$	$u_1(a,s) = \overline{u}_1(a,s)$ or $\Theta'(a) = \overline{\Theta}'(a)$	$\overline{\Theta}''(l) = 0$
3		$\sigma_{1\hat{\omega}}(a,s) = \overline{\sigma}_{1\hat{\omega}}(a,s)$ or $\Theta''(a) = \overline{\Theta}''(a)$	
4		$M + \overline{M}_1(s) = M_1(s)$ or $M - \overline{E}\overline{I}_{\hat{\omega}}\Theta'''(a) = -\overline{E}\overline{I}_{\hat{\omega}}\overline{\Theta}'''(a)$	

Example of plane frame loaded by single moment. The frame consists of two steel bars of box type with **CS** having dimensions 20x10cm and walls thickness 0.5cm, Ref 30. Properties of material are: E=205GPa, (E_1=225,27GPa), G=80GPa, $\nu = 0{,}3$. At both ends - A and C, it has torsion ($\Theta = \Theta' = 0$) and all displacements fully constrained, Fig 36a. The length of both bars is 100cm. By such conditions, were searched displacements of node B, determined analytically, too. There bar BC is almost bended, only; and bar BA mainly torsioned. Obtained results are visible in the Fig 36b. The same results are visible in Table 7.

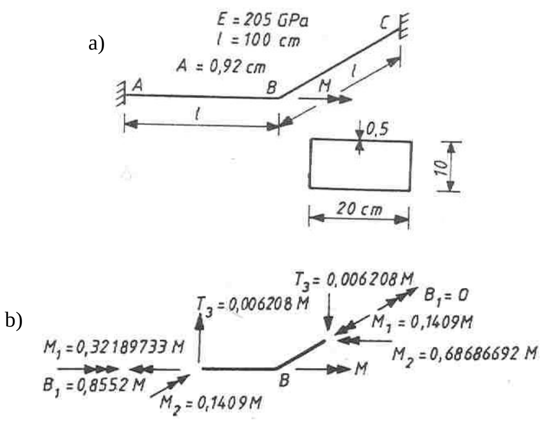


Fig. 36. Plane frame torsioned by concentrated moment, Ref 30.

Table 7. Internal forces in frame of the Figure 36, Ref 30.

Force	Solution by DMEM		Traditional solution	
	BC	BA	BC	BA
T_3	-0.006208M	0.006208M	-0.006181M	0.006181M
M_1	-0.1409M	0.32189733M	-0.1422M	0.3180M
M_2	0.68686692M	-0.1409M	0.6819M	-0.1422M
B_1	0.0000000213M	-0.8552M	---	---

So, checking equilibrium of node obtained by DMEM we see:

a) shearing forces T_3 have ideal equilibrium;

b) in direction BC bending moments are in equilibrium with torsion moment:

0.1409M -0.1409M= **0**;

c) in direction BA bending moments and torsion moments have almost good equilibrium:

1M-0.32189733M-0.68686692M= **-0.00876425M** ,

d) bimoments are not here in equilibrium:

$0.8552M \neq 0$ and $0.0000000213M \neq 0$.

This way we obtain result, that bimoments alone, are not in equilibrium in node B. This confirm thesis of the paper Ref 310, that it is no equilibrium of bimoments in node (page 399, row 4 from down).

But, if we look on physical sense of the task, we should agree, that in bar BA we have strong torsion generated by external moment and there should appear strong bimoment. Similarly in the bar BC we shell not expect of torsion. In fact, the torsion is much smaller and there bimoment almost does not exists.

In this example, we shell remember, that:

- this task have conditions proper rather for theory of second order, where node by analysis show certain deflection,
- there, was used particular bar model with planarly (fully) constrained both ends ($\Theta' = 0$ - rigid node). By numerical analysis, it is difficult by such approach to introduce for box type thin-walled bar model and node, elastically constrained displacements (here in node B, compare Refs 308, 309). Solution of this task can be done by proper definition of coefficients $\overline{C}_i$ or $\overline{C}_i'$ to formulae of the Table 5. This matter is worthy for separate paper.

- However, we shell remember, that in task of the Fig 35, the same procedure gives full equilibrium of central node.
- It should be remembered, that mechanical behaviour of structures, especially thin-walled, strongly depends on construction of nodes (boundary conditions), Ref 9.

Numerical analysis by FEM

At the end, it is necessary to comment accuracy of space bar frames analyses, by means of commercial computer systems. Generally, we can conclude, that they are to simply. Normally, in literature are reported the finite elements with six displacements per node, with only one following term in stiffness matrix, only, which describe bar torsion:

$$M_1 = \frac{\overline{G} I_1}{l}\Theta = \frac{K_s}{l}\Theta \quad . \tag{17}$$

Contrary, in the book Ref 30 was derived exact finite element for thin-walled bar on the ground of bar physical relations presented in the Table 5. There, torsion moment and bimoment depend on two coefficients related to bar torsion angle and its derivative. This approach answer at all to vector character of the task, as in example of the Fig 36.

Part of torsion by stresses calculation. In the book, Ref 30, are presented following formulae for calculation of shearing and normal stresses for composite bars:

$$\tau_{1s} = \tau_O - \frac{1}{\delta}\left(\frac{T_2\tilde{S}_3}{\overline{I}_3} + \frac{T_3\tilde{S}_2}{\overline{I}_2} + \frac{M_{\hat{\omega}}\tilde{S}_{\hat{\omega}}}{\overline{I}_{\hat{\omega}}}\right), \tag{18}$$

$$\sigma_1 = \frac{E_1}{\overline{E}}\left(\frac{T_1}{\overline{A}} - \frac{M_3\eta_2}{\overline{I}_3} + \frac{M_2\eta_3}{\overline{I}_2} + \frac{B\hat{\omega}}{\overline{I}_{\hat{\omega}}}\right). \tag{19}$$

Moreover, it is proposed to be applied additionally Huber-Mises-Hencky hypothesis, similarly as in previous Polish standards for steel structures:

$$\sigma_{red} = \sqrt{\sigma_1^2 + 3\tau_{1s}^{\ 2}} \le aR \cdot \tag{20}$$

Summary for torsion phenomena. The paper gives only short comments to principal theories – more frequently applied, to ideas, its advantages, problems and week sides, too. But it is clear, that even in so wide text and not long presentation, some topics can be reported, only.

The paper recommends taking torsion into consideration on all stages of designing process. Especially important is approach based on vectorial equilibrium of all forces acting on space frames nodes, including bimoment, too. It seems to be nowadays the most exact form of its analysis, when we use one finite element or physical relations for whole bar as its model (one bar – one finite element or one set of physical relations).

There, we can recognize following advantages:

- it is possible to be applied by any kind of analysis (static, dynamics, theory of second order, etc.,
- there, are possible declarations of the bars different boundary conditions, for four displacement functions: u_1, u_2, u_3, Θ, J.B.Obrębski, Refs 30, 32 (1991, 99),
- as result of numerical analysis we obtain as internal forces bimoments, too (responsible by torsion for significant or sometimes dominating warping stresses),
- the method (by DMEM or by FEM) enables numerical analysis of space frames or continuous beams,
- for continuous beams (e.g. bridges) obtained numerically bimoments are at all exact!; for space frames results can have certain accuracy following of problems with definition of nodes deformation,
- by numerical analysis it is the most difficult problem to take into consideration real, elastic behaviour of nodes (its deformability), but it is possible to be easy considered improving coefficients $\overline{C}_i$ or $\overline{C}_i'$ (Table 5). This will be discussed in nearest future.

3.5. Stability of structures

Instability of the **TW** or any type of the bar can be observed by any kind of loading: longitudinal compressing or even tensioning forces, by bending, by torsion moment and by bimoment, too (see Figs 10-13). There, is proposed uniform criterion for instability of any kind of structures.

3.5.1. Short review of old well known analytical methods

If we carefully observe classical, well known solutions presented from years in all academic books for Strength of Materials and for Structural Mechanics, we can come to following conclusion. In such all tasks:

- bending type of Euler's critical force e.g. Refs 30, 129, 300,
- instability of bending-torsion or torsion type, only – V.Z. Vlasov Ref 310,
- problem of frequency of free vibrations - W.Nowacki Ref 300, 301,
- task of critical loadings of simple frames - W.Nowacki Ref 301, J.B.Obrębski Fig 41C,
- dynamical stability of simple bar structures - J.B.Obrębski Fig 12 Ref 30,

as central point of calculations, was used conditions - comparing to zero main determinant of stiffness matrix or similar one, called as stability matrix of whole structure. It confirm some new, own analytical solutions which were executed applying computer methods, too.

3.5.2 Uniform criterion for instability of structures

The present chapter gives certain summary of three previous lectures presented on Structures Instability Symposia in Ref 105 (1997), Ref 129 (2000) and Ref 170 (2006) in Zakopane. As efficient criterion of structures instability is considered *comparison to zero of main determinant of whole structure - its stiffness matrix* , Eqn (21). Simultaneously, the same criterion is fulfilled when structure is geometrically changeable. In mentioned papers all examples where concerning of tasks with loading acting on given positions. Next, were shown efficient applications of this criterion to moving loadings, too Refs 238, 241,256.

Mentioned criterion has very simple form:

$$det(K)=0 \quad , \tag{21}$$

where, K is simply a main determinant of a set of equations describing equilibrium or simply in other words - stiffness matrix of the whole structure. In some approaches for particular tasks the matrix K can be built in some other ways. For example it can be built analytically (e.g. Refs 298, 300, 301, 310, 30, 45) as a stiffness matrix of FEM or composed by finite differences (e.g. Refs 235, 236, 136). On the basis of the above thesis, the following two conclusions were drawn:

1) The structure which in an unloaded state has its scheme geometrically unchangeable, where det(K)≠0, can under a certain combination of loading P with frequencies of free vibrations ω and/or given support displacements, reach a state when

$$det[K(P,\omega)]=0, \tag{22}$$

which implies the state of the instability of the structure and the possibility of obtaining a mechanism of motion, similar to geometrically changeable behaviour.

2) In each case when the main determinant of the stiffness matrix det(K)=0, it means that the structure has the possibility of reaching the mechanism of motion. For an unloaded structure it means geometrical changeability of its scheme and for a loaded, stable structure – a state of critical loading. The Eqn (22) is a particular case of criterion (21).

The conclusions described above are valid for the problems of:

- any kind of analysis: static, dynamics, stability and dynamical stability,
- any type of loading: static or dynamical, with any kind of structure interaction with the external media,
- any type of analysis: - analytical solutions of equilibrium equations, - analytical solutions of finite differences equilibrium equations, - in numerical displacements methods: of FEM (Finite Elements-), FDM (Finite Differences-), DMEM (Difference Matrix Equations-) or 3D-TSM (3 Dimensional and Time Space Method).

Now, we can revise well known solutions in the light of conditions - Eqns (21, 22) and then present own tests. It can be enumerated the simplest tasks, starting from Euler's, through bending-, bending-torsion and torsion, only, types of instability, from single straight bars to critical loadings of large space bar structures. In the same way various types of tasks for dynamical instability of bridges under moving loading (cars, aircrafts) were considered. Now we can say, that such solutions were obtained using the well known 3D-TS method. Examples of numerical results for some of the author's own tests are given below.

General remarks to application of uniform criterion for instability of structures. It can be shown some examples of application, of the general, uniform criterion of structures instability – Eqn (21, 22). In all calculated examples, applying FEM, FDM, DMEM and 3D-TSM, this condition was giving enough exact result. It seems that this condition is not only necessary, but sufficient, too. There is possible to calculate: first or higher values of critical forces or critical sets of forces; modes of instability for investigated structures – its shape in all considered time moments; associated with deformations internal forces, bending-, bending-torsion or only torsion type instability, etc.

The method can be applied to wide class of tasks concerning static, dynamics, stability, dynamical stability concerning various structures, including composite ones.

3.5.3. Determination of critical force using Finite Differences

There, various approaches to numerical application of FDM are possible, but the equilibrium equation of the whole structure always has the shape $Kx = Q$, where: K - stiffness matrix of the whole structure, x - vector of node displacements, Q -vector of external loadings. It is always a set of linear algebraic equations. Its solution belongs to elementary numerical tasks. By the process of unknowns *x* determination, using Gaussian eliminations, the value of D=det(K) can be additionally (by the way) calculated.

By this description determination of critical forces follows condition (21). There we look for the value of the determinant of stiffness matrix D=det(K)=0. In general, it depends on the values of some chosen variable parameters. So, critical combined loading can be different by certain combinations of some parameters and shall fulfill the condition as below:

$$det[K(P,\omega,v,a,M,m,d,t)]=0 \quad , \tag{23}$$

where: P – system of one or more forces, ω – frequency of free vibrations, v – loading velocities, a –acceleration of loadings, M – moving masses, m –mass of structure, d – dumping conditions, t –time etc. The conditions (21), (22) are particular cases of Eqn (23). It was efficiently tested, that the stiffness matrix can be composed on the basis of more than one differential equation (finite differences operators).

3.5.4. Instability of bars under combined loading

Especially important is problem concerning of bars under action external combined loadings, which are loaded by all kinds of forces: longitudinal, transversal, bending moments, torsioning and even bimoments. As it is shown in the books Ref 30, 35, application of uniform criterion – comparison to zero main determinant of stiffness matrix to the bar, or to structure Refs 105, 129, 170, 256, is very efficient (all by J.B.Obrębski). Next, some such numerical tests were done by J.Tolkstorf Refs 168, 176 where are shown certain ultimate curves or surfaces for critical sets of two or more combined loadings.

On the ground of conditions (21), (22) and (23) can be determined combinations of critical external loadings including given boundary conditions, by means of analytical methods or by FDM or even by MathCAD application.

In result were find diagrams of ultimate critical bar loading (for q_y ,P), as e.g. in the Fig 37, or ultimate critical surfaces (for P_1, q_2 , q_3) as in the Fig 38.

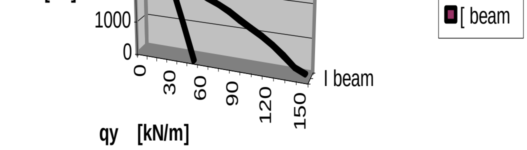
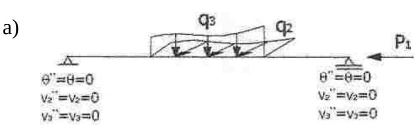


Fig. 37. Ultimate critical curves of forces for simply supported beam with I cross-section (at axes 145.4 and 2161 - lower) and with channel cross-section (at axes 48.559 and 3386 - higher)

a)



b)

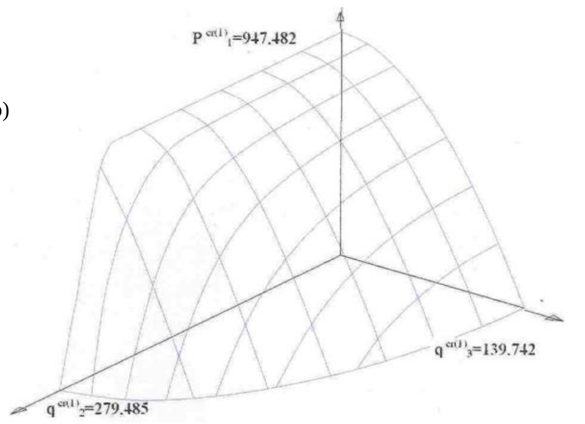


Fig.38. b) Ultimate critical surface by given boundary conditions and scheme of the Fig 36a

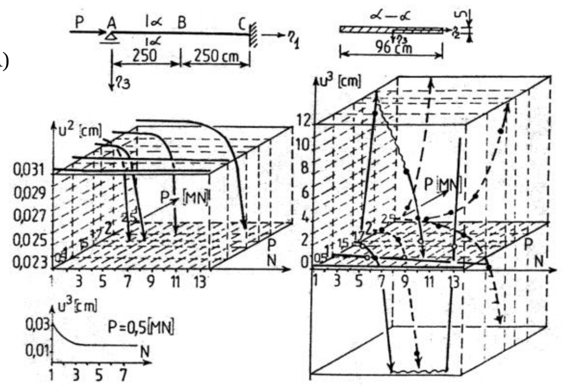
3.5.5. Examples of uniform criterion application

In the paper Ref 105 where pointed some other methods of evaluation geometrical unchengeability of structures by: J.Karczewski and J.König or by J.B.Obrębski. They will be here not repeated.

Examples of numerical results for some own tests, are given below.

In example from Fig.39A, were additionally observed transversal displacements u^2 or u^3.

A)



B)

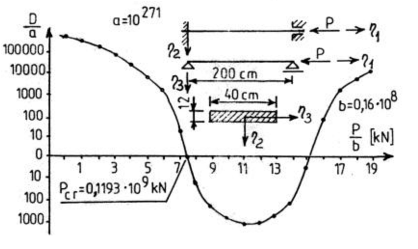


Fig. 39. A) Displacements for middle point (B) of compressed bar; B) Values of D=det(K(P)), for compressed bar. Buckling in plane $0\eta_1\eta_2$; both Refs 11, 236, 170.

Next in the book Ref 30 was presented example determination of dynamical stability of the straight, freely supported bar, Fig 40, based on three differential equilibrium equations of its axis, applying condition (22). The diagram at right side of Fig 40 was made on computer changing parameters P and ω.

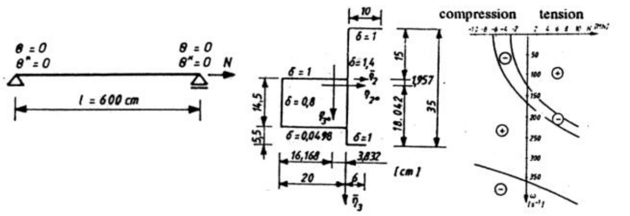
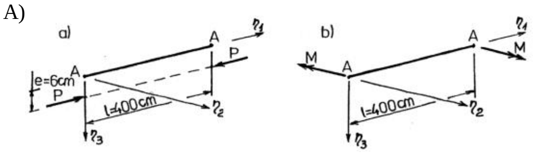


Fig. 40. Critical forces and values of free vibrations for freely supported bar Refs 30, 105.

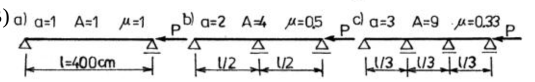
Next, in the Ref 129 where given analytical examples for composite bars and composite frames, based on theory contained in the books Refs 30, 35. There as condition was applied identically as in V.Z. Vlasov's book, equation (21), built on the basis of three differential equilibrium equations of bar axis, Figs 41A. Additionally, the examples were extended on bar boundary conditions shown in the Fig 41B. Geometrical characteristics of four composite **CS**s, Figs 20b-e, were calculated accordingly to theory proposed by J.B.Obrębski in the book Ref 35 and in some papers – e.g. Ref 235.

Further, in the paper of Ref 129, was given the analytical example of critical loading determination, for simple frame shown in the Fig 41C. There, results were compared for five similar frames, but built of bars with homogenous and composite **CS**s, Fig 20, by symmetrical and unsymmetrical instability modes.

A)



B)



C)

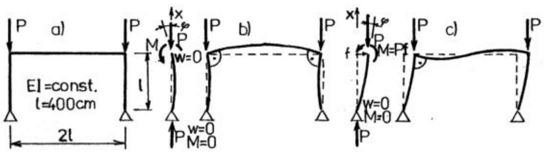
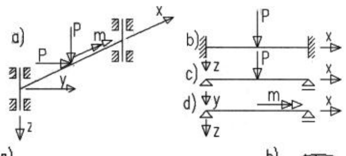


Fig. 41. A) Schemes of eccentrically loaded and purely bended bars Refs 310, 30, 236. B) Different bars boundary conditions Refs126, 236. C) Frame and assumed deformation modes Refs 126, 236.

It was efficiently tested, that stiffness matrix can be composed on basis of more than one, differential equations (***F****inite* ***D****ifferences* ***O****perators*). Such test for three equations on one bar, Fig 42A was shown in Ref 146. It is similar to method used by J.B.Obrębski for space bar structures, called *Difference-Matrix Equation Method* (DMEM), Ref 11.

Example of calculation of critical force applying FDM was presented e.g. in Refs 235, 238, where wooden bar was divided on 10 sections, Fig 42B.

A)



B)

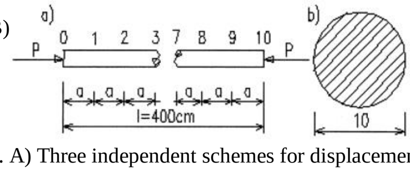


Fig. 42. A) Three independent schemes for displacements of single bar Ref 146, 238. B) Axially compressed pine bar, Refs 146, 235, 238.

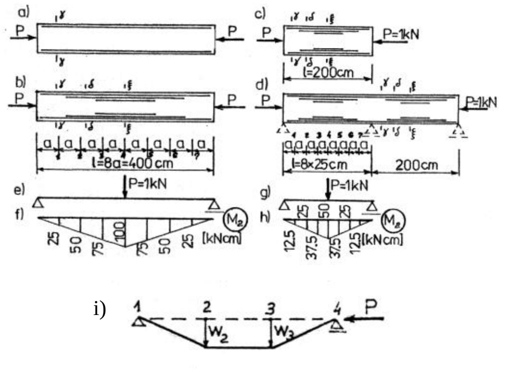


Fig. 43. Reinforced concrete compressed bars and assumed bending scheme Refs 232, 238.

The other example, concern the bar with variable rigidity (three zones of reinforcement). It was solved using linear finite differences operator and Vianello method, Figs 43a-d. It is possible to calculate the critical force P, using criterion (21), too. There can be applied operators of first or second order theory. Similarly, for example from Fig 43i, critical force can be determined in three ways Ref 35. The first, as above, built with linear, first order operator by means of Vianello method Ref 35.

The second, applying the same equations and moving right side terms with loadings to the left side ($Kx - Q = 0$), we come to condition (21) Ref 35. The third can use the mathematical approach of searching eigenvalues of the same equations.

3.5.6. Determination of critical force using FEM

Wide explanation of the ***F****inite* ***E****lement* ***M****ethod* application, was in book Ref 312 given. There, as central point of the method is proposed comparison to zero of main determinant, of the whole structure, so called, general stability matrix (using theorem of Lagrange-Dirichlet). This approach seems to be much more complicated with comparison to the criterions (21, 22) presented here. It will be not wider discussed.

3.5.7. Determination of critical loading using 3D-TSM

The idea is not new. It was applied e.g. in the papers of Z. Kączkowski, M.Witkowski, and A. Podhorecki (all Poland), and probably by the others. In that papers were used different *Time-Finite Elements* approaches, concerning of very simple structures, only.

In the works of J.B.Obręski and R.Szmit (wider literature see in Refs 109, 115, 120, 122, 124, 128, 133, 149, 223), in general case, four *differential equilibrium equations* as the function of time and three unknown linear displacements and rotation of the bar (with regard to its longitudinal axis), were formally replaced by similar four *Finite Differences Operators* of Boolean type. There is assumed existence of four-dimensional space; where besides of three dimensions used in Euclides'ian space, the time is the fourth dimension. So, we get **3D-Time Space**. At the boundaries of considered space, should be applied modified sets of equations, Ref 223. Such four equations written for each point of division and for each time moment (point on time axis) are giving ***D****ynamical* ***S****tiffness* ***M****atrix* K. From mathematical point of view it gives set of *linear algebraic equations* (23). Looking for critical loading, we calculate D=det[K(P,ω,v,a,M,m,d)]. The quality of obtained results depends on the choice of assumed space division – specially, on the time step which should be carefully selected. Generally, we can conclude, that the method shows high accuracy of obtained results – compared with some known in world literature examples. There is possibility to calculate displacements and internal forces, too.

3.5.8. Part of instability and torsion by stresses calculation

Is possibility of very easy and simultaneously exact calculation of critical loading of any type bars, under action of longitudinal and shearing forces, by moments and even by bimoment. Therefore, at all is right proposal by Obręski, to dimensioning bars **CS**s using four instability coefficients m_{wi}, for each of four internal forces, including bimoment, (Refs 190, 191, Singapore CI-PREMIER (1990) and Ref 30, 32 (1991, 1999)) as in formula:

$$\sigma_1 = \frac{E_1}{E}\left(m_{w1}\frac{T_1}{A} - m_{w3}\frac{M_3\eta_2}{I_3} + m_{w2}\frac{M_2\eta_3}{I_2} + m_{w4}\frac{B\omega}{I_\omega} \right), \tag{24}$$

or even one m_w as below, only:

$$\sigma_1 = \frac{E_1}{E} m_w \left(\frac{T_1}{A} - \frac{M_3\eta_2}{I_3} + \frac{M_2\eta_3}{I_2} + \frac{B\omega}{I_\omega} \right). \tag{25}$$

In both cases it was proposed to apply additionally Huber-Mises-Hencky hypothesis, Eqn (20), similarly as in one of the previous Polish standards for steel structures e.g. PN-90/B-03200 (see Refs 30, 35 Obręski (1991, 1997)).

Moreover, the coefficients increasing values of stresses by instability, can be calculated in uniform manner accordingly to well known formula, Refs 298, 30, 35:

$$m_{wi} = \frac{P_i n_w}{P_i^{cr} n} , \tag{26}$$

where: P_i – internal force, P_i^{cr} – critical force for particular bar, n_w , n - safety coefficients by instability and by force action. Above formulae do not need more explanations, Ref 35.

4. GEOMETRY OF SOME SPACE STRUCTURES

Below, are considered three essential descriptions important for forming numerical algorithms. First, concerning of applied spaces and two approaches to structure geometry description, [1-6,8,10,12-14,17,21-24].

4.1. Applied Nets of Points – Considered Spaces

Each of described theories, by numerical implementation needs another algorithm. It is interesting, that almost in all mentioned below programs were applied identical principles for input data. There, the global geometry of whole task (space bar structure or bar cross-section, only, etc.) can be described by means of Boole's displacement operators, which for *3D-Time Space* has the form (Figs 44-46, 54):

$$E^{a_1,a_2,a_3,a_4}\Phi(x_1,x_2,x_3,t) = \Phi(x_1+a_1, x_2+a_2, x_3+a_3, t+a_4) \tag{27}$$

where: Λ - number of operation (or bar direction), $a_{\Lambda i}$ - operator's exponents; both are integer numbers; t – is the time. For static tasks it can have simplified form as e.g. for 3D, Fig 44:

$$E_\Lambda \Phi = E_\Lambda^{a_{\Lambda i}}\Phi(x_i) = E_\Lambda^{a_{\Lambda 1},a_{\Lambda 2},a_{\Lambda 3}}\Phi(x_1,x_2,x_3) = \Phi(x_1+a_{\Lambda 1}, x_2+a_{\Lambda 2}, x_3+a_{\Lambda 3}) , \tag{28}$$

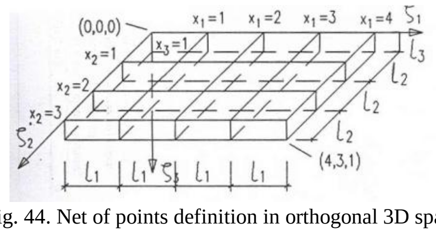


Fig. 44. Net of points definition in orthogonal 3D space

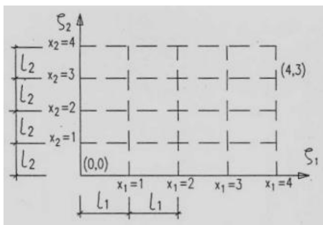


Fig. 45. Net of points definition in orthogonal 2D space, only.

Geometry of Structure in 3D, only

Position of nodes in 3D space can be declared for the simplest orthogonal net of points, by short formula:

$$\zeta_i = x_i l_i \, , \qquad (29)$$

where: ζ_i - global structure coordinates, $i=1,2,3$, x_i – integer numbers, and l_i – modules of net of points – real numbers.

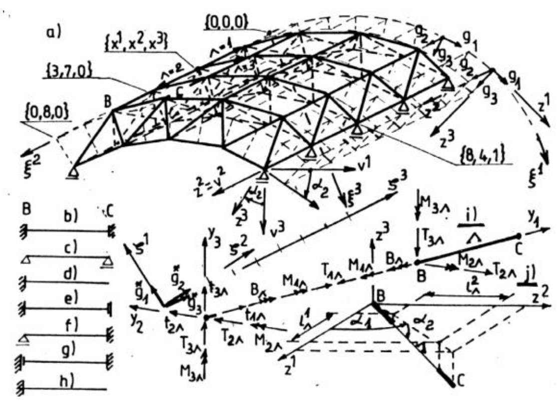


Fig. 46. Definition of more important elements of space bar structures description.

Examples of nets for 2D (i=2) and 3D (i=3), are shown in the Figs 45, 44. Definitions of similar, more complicated nets of points are given in the Refs 11, 14, 15, 184, 200, 204, 217, 233, 279 and in Figs 48-52.

In program system *WDKM*, as theoretically the most advanced, are used six essential coordinates systems. The first, global ζ_i as e.g. in Eqn (29) and three next:

z_i - local coordinate system in which is considered equilibrium of nodal forces and are calculated node displacements Fig 46a,

$\eta_i = y_i$ -coordinates for particular element (bar principal, reduced axes), with regard to it, are considered nodal bar internal forces,

v_i - support coordinat*e system* – for definition of given planes of each support shifting, Fig.46a.

Definition of elements the structures description, explain the Fig.46. Two additional coordinates will be shown in the chapter 4.3.2.

4.2. On the geometry of plane hexagonal grids

In the works Refs 1-4, were investigated plane single layer grids and double layer hexagonal trusses.

They were inscribed into inclined net of points, with angle 120^O between axes, Fig 29. The repeatable nodes were of one type, only. Node is connecting three bars along which were assumed three directions Λ=1,2,3 and three Boolea's operators Eqn (28). Rotation of node by 180^O was obtained thanks assumption of functional character of exponent $a_{\Lambda i}$ of Boole's operators (Eqn (28)) and by introduction special mathematical operations (new elements of mathematic). In result, was obtained possibility of analytical solutions.

The Fig 47 shows the biggest regular bar pattern with regular, which can be inscribed exactly in circle. Each smaller can be inscriber in smaller circles, too.

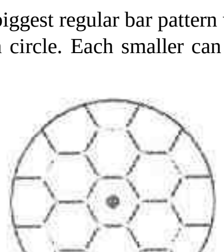


Fig. 47. Maximal regular hexagonal grid inscribed into circle, Ref (4).

4.3. Geometry of large space bar structures.

Next step, after analysis of relatively simple space bar structures – as hexagonal ones, was theory prepared for any kind of space bar structures and its application in original own computer programs. As fundamental element of such theories, is geometrical description of such objects.

4.3.1. Geometry, examples and architectural aspects of the family of two-curvature space bar structures.

The geometry for such type of structures was consequently derived by means of vector calculus, Ref 11. There, were elaborated detailed mathematical relations and geometrical objects for following kinds of net of points:

- orthogonal net of points,
- translational net of points,
- rotationally-translational net of points,
- barrel net of points,
- cylindrical structures,
- ring net of points,
- spherical net of points,
- conical net of points,
- toroidal net of points.

Next it was extended on two, wery important nets of points for:

- elliptical structures,
- wavy structures.

In all listed above cases, description starts from definition of the bar orientation in given net of points. Next, should be assumed the most convenient local node coordinates and operations on vectors defined in this coordinates. After preparation of such elements of analytical geometry, can be built whole numerical algorithm and programs, using additionally proper equilibrium equations of repeatable node. More detailed information on the matter, can be found in the work Ref 11.

Below, are provided definitions, certain essential explanations and computer drawings of different forms, which should be very useful for civil engineering large scale coverings – domes, wavy domes, barrel and cylindrical vaults etc. It presents some information on previous author's works concerning of computer graphics, analysis and synthesis of complicated space bar structures. There, description of structure global geometry and detailed topology is defined by introduction in input data the following groups of information:

- kind of applied net of points,
- list of nodes inscribed into declared net of points,
- definition of repeatable nodes,
- definition of repeatable bars,
- given support system,
- external loading.

Some general assumptions of elaborated theories and programs, are explained a little in presented below Figs 48-52

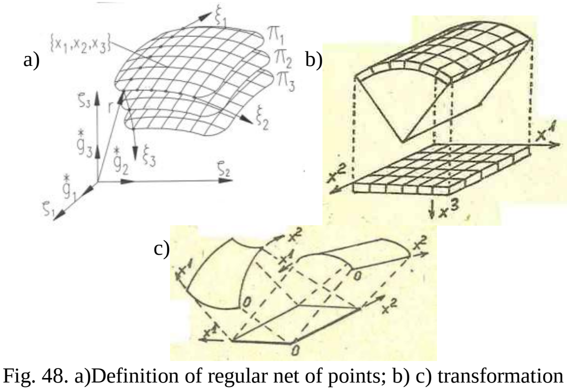


Fig. 48. a)Definition of regular net of points; b) c) transformation of orthogonal coordinates x_i into curved space (surfaces).

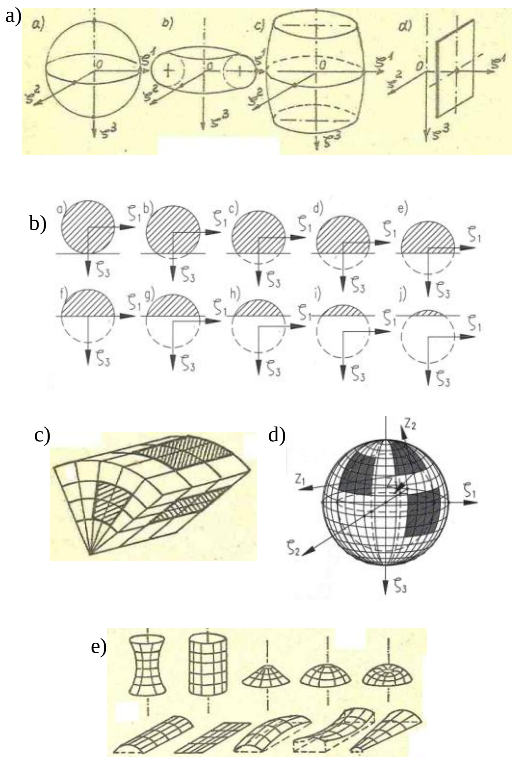


Fig. 49. Possibilities to obtain various architectural effects by: a) choice of global shape of nets of points; b) choosing other parts of spherical net of points; c) d) location of structure on other parts of net of points; e) cutting convenient part of selected nets of points.

It is assumed, that all nodes of structure are inscribed into regular net of points located in intersections of two, in general case, curved parametrical lines lying on some equi-distance smooth surfaces, Fig 48a. There, are also explained general assumptions of structure geometry and some its detailed relations and formulae helpful by computer analysis of double-curvature bar structures (Figs 48-52. The architectural and mechanical aspects of considered objects are pointed, too.

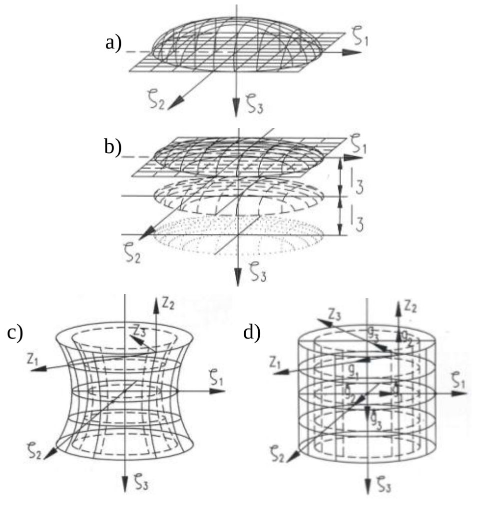


Fig. 50. Definitions of different kinds of translational net of points

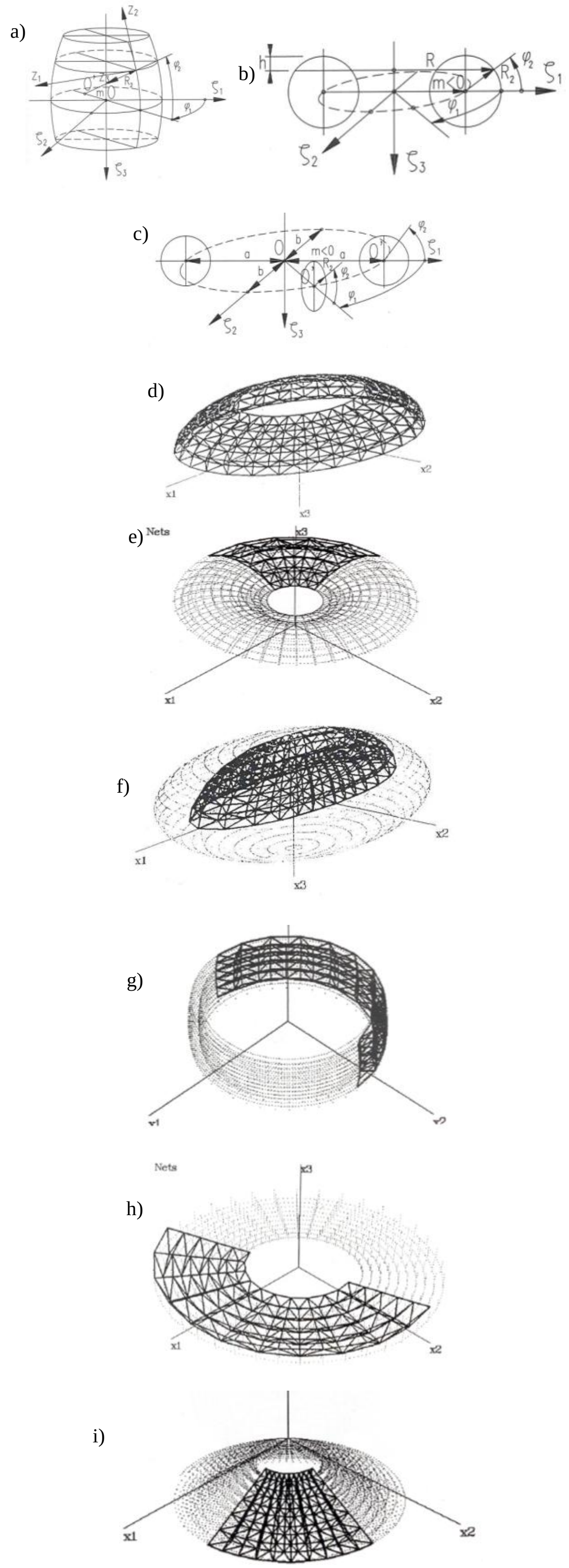


Fig. 51. Definition of nets of points: a) double curvature revolution type; b) toroidal (obtained from case a)); c) elliptical (modified net a)); d-i) applications for: double-layer spherical-, toroidal-, elliptical-, barrel-, ring-, and conical type bar structures

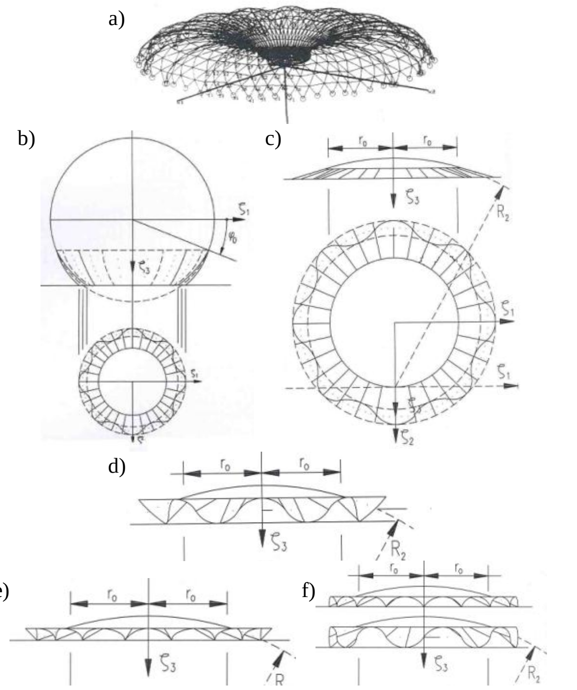


Fig. 52. Examples of wavy space bar structures: a) wavy toroidal, b) wavy supporting part for sphere, c) horizontal waves, d) horizontal-vertical waves, e) absolute horizontal-vertical waves, f) vertical waves. The waves starts from certain radius r_o.

Described shaping of structures is accessible in two program systems WDKM and SPES. Moreover, there is possible numerical analysis, synthesis and optimization.

Additional assumptions for geometry of space bar structures . By description of structure geometry and detailed its configuration, are used six coordinates Refs 11, 14, 15, 28,29 etc.:

1. global orthogonal ζ_i , with vector basis $\vec{g}_i^{\,*}$, Figs 44-46, 48-52,
2. local coordinates orthogonal z_i with vector basis $\vec{g}_i^{\,z}$, for calculation of nodes displacements and equilibrium or motion equations, $\vec{r}_z = z_i \vec{g}_i^{\,z}$, Figs 49-51,
3. supports orthogonal coordinates v_i with vector basis $\vec{g}_i^{\,v}$ for description of its inclination $\vec{r}_v = v_i \vec{g}_i^{\,v}$,
4. parametrical coordinates x_i for identifying of node positions in coordinates ζ_i , Fig 48,
5. parametrical curvilinear lines ξ_i , Fig 48,
6. elemental (bar) orthogonal coordinates η_i for calculate internal forces for each bar, $\vec{r}_e = \eta_i \vec{t}_{\Lambda i}$.

In four above cases coordinates are real type variables: $\zeta_i \in R^3$, $z_i \in R^3$, $v_i \in R^3$, $\eta_i \in R^3$. Only parametrical coordinates x_i are integer type numbers.

There, are unique functions defining global nodes positions in R^3 and vector basis of the local coordinates:

$$\psi^i : (x_i, \vec{g}_i^{\,*}) \to \vec{g}_i^{\,z} \quad \text{and} \quad \vec{g}_i^{\,z} = \beta_{ik} \vec{g}_k^{\,*}$$

or in matrix form

$$g^z = \beta \overset{*}{g} \cdot \tag{30}$$

For structure description was used Boole's operator Refs 1-5, 8, 11, 12, 30 etc., directed along the bar with number Λ connecting two nodes A and B as in the figure 54, Eqn (28).

In all nodes of particular net of points the local coordinates z_i are defined identically. There were used three types of its orientation:

1. The axes $z_i \parallel \zeta_i$ - parallel, possible to be applied for all kinds of nets of points.
2. The axes: z_1 - tangent to meridians of rotational nets of points; z_2 - tangent to parallels; z_3 - orthogonal to both previous, (see Fig 49d).
3. Cylindrical type as in the Fig 50, similar to type 2, but $z_2 \parallel \zeta_3$.

In this coordinates system are defined external loadings and calculated unknown nodes displacements. So, the choice of orientation of this coordinates was left for user of program.

For adding two vectors of forces or displacements given in two nodes A and B connected by bar Λ (see Fig 54), is needed operation:

$$E_\Lambda \vec{u} = \vec{g}_i \bullet A_{\Lambda in} E_\Lambda u_n \ ,$$

where, was defined as matrix scalar operation geometrical object:

$$A_{\Lambda in} = \vec{g}_i \bullet E_\Lambda \vec{g}_n \ .$$

The same in matrix description (31) can be written as follow:

$$A_\Lambda = g^z \left(E_\Lambda g^z\right)^T = \beta \overset{*}{g} \bullet \overset{*}{g}^T E_\Lambda \beta^T = \beta\left(E_\Lambda \beta^T\right) \ . \tag{31}$$

For orthogonal net of points when $z_i \parallel \zeta_i$ (type 1): then $\beta = I$ and $A_{\Lambda in} = \delta_{\Lambda in}$ - it is delta of Kronecker.

In case of double-curvature revolution net of points (type 2) we have:

$$\beta = \begin{bmatrix} -s_1 & c_1 & 0 \\ -c_1 s_2 & -s_1 s_2 & -c_2 \\ -c_1 c_2 & -s_1 c_2 & s_2 \end{bmatrix}$$

$$A_\Lambda = \begin{bmatrix} \bar{c}_1 & -\bar{s}_1 \bar{s}_2 & -\bar{s}_1 \bar{c}_2 \\ \bar{s}_1 s_2 & \bar{c}_1 s_2 \bar{s}_2 + c_2 \bar{c}_2 & \bar{c}_1 s_2 \bar{c}_2 - c_2 \bar{s}_2 \\ \bar{s}_1 c_2 & \bar{c}_1 c_2 \bar{s}_2 - s_2 \bar{c}_2 & \bar{c}_1 c_2 \bar{c}_2 + s_2 \bar{s}_2 \end{bmatrix} ,$$

$$\bar{s}_2 = \sin(\varphi_2 + \alpha_2) \ , \quad \bar{c}_2 = \cos(\varphi_2 + \alpha_2) \ , \quad \alpha_\beta = a_{\Lambda\beta} \Delta\varphi_\beta \ .$$

For cylindrical revolution net of points (type 3) we find:

$$\beta = \begin{bmatrix} -s_1 & c_1 & 0 \\ 0 & 0 & -1 \\ -c_1 & -s_1 & 0 \end{bmatrix} , \quad A_\Lambda = \begin{bmatrix} \bar{c}_1 & 0 & -\bar{s}_1 \\ 0 & 1 & 0 \\ \bar{s}_1 & 0 & \bar{c}_1 \end{bmatrix}$$

$$s_\alpha = \sin\varphi_\alpha \ , \quad c_\alpha = \cos\varphi_\alpha \ , \quad \varphi_\alpha = x_\alpha \Delta\varphi_\alpha \ .$$

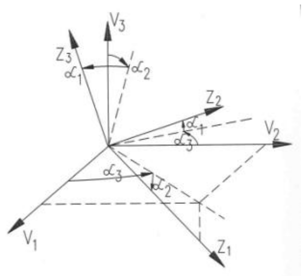


Fig. 53. Definitions of rotation angles between coordinates z_i – local and v_i – of support.

$$z = Bv \ , \qquad B = \begin{bmatrix} c_2 c_3 & c_2 s_3 & -s_2 \\ s_1 s_2 c_3 - c_1 s_3 & s_1 s_2 s_3 + c_1 c_3 & s_1 c_2 \\ c_1 s_2 c_3 + s_1 s_3 & c_1 s_2 s_3 - s_1 c_3 & c_1 c_2 \end{bmatrix} , \tag{32}$$

where: $s_i = \sin\alpha_i$, $c_i = \cos\alpha_i$, Fig 53.

Moreover, are valid transformations between coordinates:

$$\eta = \overset{o}{B} z \ ,$$

where $\overset{o}{B}$ is identical with B when: $s_i = \sin\beta_i$, $c_i = \cos\beta_i$

$i=1, 2, 3$ and (see (32) and figure 54)

$$\eta = \overset{o}{B} Bv = \overline{B}v \ .$$

There, are valid reverse transformations:

$$v = \overset{\vee}{D} z \ , \quad z = \overset{o}{D} \eta \ , \quad v = \overline{D} \eta \ ,$$

where: $D = B^T \ , \quad \overset{o}{D} = \overset{o}{B}{}^T \ , \quad \overline{D} = \overline{B}{}^T \ .$

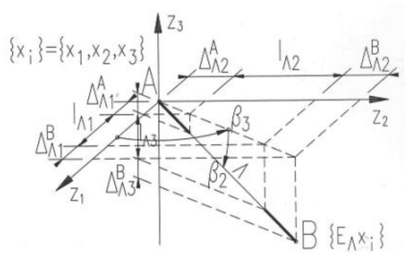


Fig. 54. Definition of angles β_i for bar Λ inclination, (rigid nodes as in the Fig. 21).

For elemental (bar) orthogonal coordinates η_i the unit basis vectors are defined as follow:

$$\overset{0}{D} = \overset{o}{B}{}^T = \left[\{ t_{\Lambda 1} \} \quad \{ t_{\Lambda 2} \} \quad \{ t_{\Lambda 3} \} \right] \ .$$

The unit vectors $\overrightarrow{t_{\Lambda i}}$ are oriented along of longitudinal bar axis and along two principal axes of bar cross-section. Angle β_1 as rotation of the bar round the longitudinal bar axis is defined in input data. The two remaining angles - β_2 and β_3 are calculated accordingly to Fig 54 by formulae:

$$\sin\beta_2 = -\frac{l_{\Lambda 3}}{l_\Lambda} \ , \quad \cos\beta_2 = \frac{l'_\Lambda}{l_\Lambda} \ , \quad \sin\beta_3 = -\frac{l_{\Lambda 2}}{l'_\Lambda} \ , \quad \cos\beta_3 = \frac{l_{\Lambda 1}}{l'_\Lambda} \ ,$$

$$l'_\Lambda = \sqrt{(l_{\Lambda 1})^2 + (l_{\Lambda 2})^2} \ .$$

Projections of the bar Λ on axes z_i depends on kind of applied net of points and on orientation of axes z_i (types 1, 2 or 3):

For orthogonal net of points (type 1 of z_i): $l_{\Lambda i} = l_i a_{\Lambda i}$.

For double curvature revolution structures (type 2 of z_i):

$$l_{\Lambda 1} = -r'_2 A_{\Lambda 13} \ , \quad l_{\Lambda 2} = -r'_2 A_{\Lambda 23} \ , \quad l_{\Lambda 3} = r_2 - r'_2 A_{\Lambda 33} \ ,$$

$$r_2 = R_2 - x_3 l_3 \ , \quad r'_2 = R_2 - (x_3 + a_3) l_{\Lambda 3} \ .$$

For cylindrical structures (type 3 of z_i):

$$l_{\Lambda 1} = -r'_1 A_{\Lambda 13} \ , \quad l_{\Lambda 2} = l_2 a_{\Lambda 2} \ , \quad l_{\Lambda 3} = r_2 - r'_2 A_{\Lambda 33} \ ,$$

and $r_1 = R_1 - x_3 l_3 \ , \quad r'_1 = r_1 - a_{\Lambda 3} l_3 \ .$

For spherical structures (type 2): it is particular case of double curvature revolution structures, where:

$$r_1 = r_2 = R_2 - x_3 l_3 \ , \quad r'_1 = r'_2 = R_2 - (x_3 + a_3) l_{\Lambda 3} \ .$$

For any case of net of points, including above and wavy structures, there is possible to regard, that local coordinates are parallel to global ones (type 1 of z_i) and than, we simply have:

$$l_{\Lambda 1} = \zeta_1^B - \zeta_1^A \ , \quad l_{\Lambda 2} = \zeta_2^B - \zeta_2^A \ , \quad l_{\Lambda 3} = \zeta_3^B - \zeta_3^A \ .$$

4.3.2. Geometrical fundations and architectural possibilities of UNIDOM space bar system.

Contrary to family of spherical domes, commented in previous chapter, for UNIDOM structures are presented investigation results of its global geometry and bar pattern, only. Up to this time, detailed elements of mathematical description of nodes and particular nodes, helpful for composition of equilibrium equations, were not elaborated.

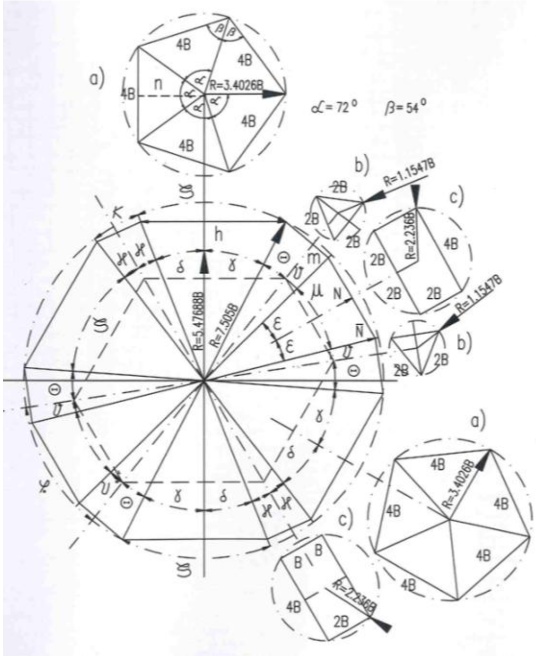


Fig. 55. Detailed geometry of rhombiicosidodecahedron.

Basic parameters concerning the geometry of domes of polyhedron type, forming UNIDOM (UNIfied DOMes) space bar system, are elaborated. Therefore, below are shown formulae and drawings explaining in details geometry of such structures and possible architectural effects. In all cases the designed domes can be composed by application of very limited number of bars and nodes. So, the structure should be cheap and easy in prefabrication, but simultaneously variety of different architectural outlooks can be almost infinite. The possibilities proposed here, are much wider as e.g. in well known Unibat or Unistrut or Mero systems. Here is very important question, about global dimensions of whole structure, and detailed angles, dependently on the dimensions of one rectangular wall a×b. When a=4B is side of pentagon. The B means the length of "blue" bar. There, were calculated the thicknesses of double-layer substructures: h=0.5B for rectangular substructure and for irregular sector of pentagon (Fig. 2C) and h=0.6454972244B for triangular substructures and for two different sectors of pentagonal substructure (Fig. 2C).

a)

b) …….

c)

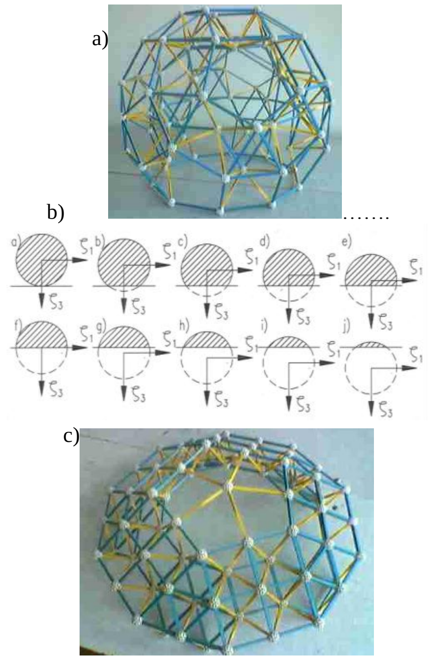


Fig. 56. Models and shaping of rhombiicosidodecahedron

Calculated **for rhombicosidodecahedron**, essential dimensions and angles Figs 55, 56; R=7.505 B, are as follow:

pentagon, Fig 55a, when a=4B (B – length of blue bar): R=3.4026B,

$$\sin\gamma = \frac{3.402603233B}{R}\ ,\ \gamma = \arcsin\left(\frac{3.402603233\,B}{R}\right),$$

$$n = 3.402603233\,B\cos 36^{\circ} = 2.752763841B\ ,\ tg\delta = \frac{2.752763811\,B}{R\cos\gamma},$$

$$\delta = arctg\left(\frac{2.752763811\,B}{\sqrt{R^2-(3.402603233B)^2}}\right)\ ,\ \zeta = \gamma+\delta\ ,$$

$$\gamma = 26.96063187^{o}\ ,\ \delta = 21.51782409^{o}\ ,\ \zeta = 48.47845596^{o}\ .$$

Triangle, Fig 55b; R=1.1547B,

$$\overline{R} = 1.154700538B\ ,\ m = 0.5773502692B\ .$$

$$\Theta = \arcsin\left(\frac{1.154700538\,B}{R}\right)\ ,\ \upsilon = \arcsin\left(\frac{0.5773502692\,B}{R}\right)\ ,\ \overline{\Theta} = \Theta+\upsilon\ .$$

Rectangle, Fig 55c; R=2.236B,

$$B = \frac{b}{2} = R\sin\overline{\kappa}\ ,\ \overline{N} = R\cos\overline{\kappa}\ ,\ N = \overline{N}\cos\varepsilon\ ,$$

$$\mu = \varphi_o - 2\upsilon\ ,\ \varepsilon = \frac{\mu}{2}\ ,$$

$$\kappa = arctg\left(\frac{B}{R\cos\overline{\kappa}\,\cos\varepsilon}\right),\ \mathrm{K} = 2\kappa\ ,\ \mu = 30.73400322^{o},$$

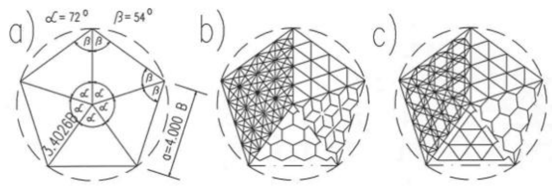
$$\varepsilon = 15.36700161^{o}\ ,\ \kappa = 7.937471229^{o}\ ,\ \mathrm{K} = 15.87494246^{o}$$

Calculated dimensions and angles (a=4B): **for dodecahedron (**K=0 and $\upsilon = 0$**):** R=5.4768804 B, $\gamma = 38.40869235^{o}$, $\delta = 30.17317768^{o}$,

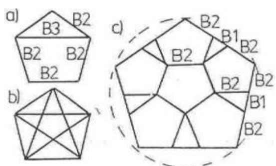
$\zeta = 68.58187003^{o}$ and angle $\varphi_o = 2\arcsin\left(\frac{2B}{R}\right) = 42.83626079^{o}$.

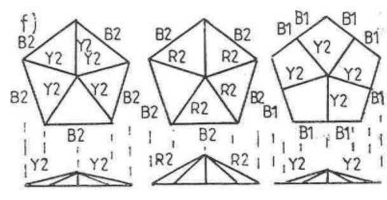
Up to the moment were discovered some possible bar patterns for flat double-layer pentagonal substructures, Fig 57A. Some next for single-layer substructures are shown in the Fig 57B.

A)



B)





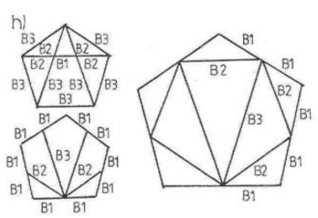
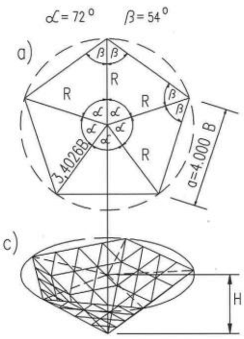


Fig. 57. Possible bar patterns for pentagon, by Zometool elements.

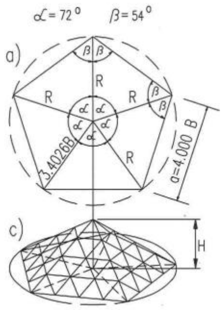
A)



h=0.645

=-2,202B

B)

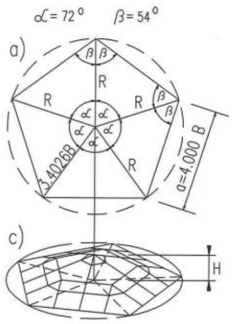


h=0.5

=1,701B

Fig. 58

C)

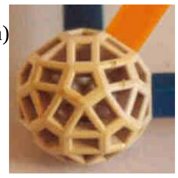


h=0.645

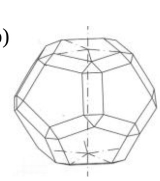
=0,525B

Fig. 58. Global geometry of: A) Concave pentagon with possible bar patterns B) Convex – high possible arrangements by ZOMETOOL elements, C) Convex - low double-layer substructures.

a)



b)



c)

Fig.e 59. Geometry of node: a) node used in Zometool kit; b) Rhombiicosidodecahedron as a model of node; c) Comparison of the nodes of rectangular and triangular substructures.

Table 8. Numbers of bars connected in node.

Type of node (polyhedron)	Blue Rectangle	Yellow Triangle	Red Pentagon
Rhombiicosidodecahedron	30	20	12

Table 9. Inclination of all possible connections in node for B-blue, R-red and Y-yellow bars.

α	Angle β [deg]																
deg	**0**	20	33	37	57	59	60	70	**90**	110	120	121	123	143	147	160	**180**
0	B		R					Y	B	Y					R		B
20	B					B						B					B
30									R								
45							Y				Y						
57	B			B				B		B				B			B
70									Y								
90	B	Y			R				B				R			Y	B
110									Y								
123	B			B				B		B				B			B
135							Y				Y						
150									R								
160	B					B						B					B
180	B		R					Y	B	Y					R		B

Table 10. Comparison of the thickness h of double-layer substructures.

	Type of substructure				
	Rec-tan-gular	Triangular	Sector of pentagon as regular triangle Fig.58A	Irregular sector, Fig 58B	Irregular sector Fig 58C
h	0.5B	0.6454972244B	0.6454972244B	0.5B	0.6454972B

Table 11. Elevation of central node in pentagonal substructure (a=4B)

	Type of pentagonal substructure		
	Concave, Fig 58A	High convex, Fig 58b	Low convex, Fig58C
H	-2.202B	1.701B	0.525B

Moreover, it was recognized geometry of node for UNIDOM space bar system, Fig 59, Ref 180, with two angles horizontal and vertical for defining inclination of the particular bars in 3D space. Detailed derivations, formulae, further drawings, remarks, conclusions and list of references are given in the paper given in mentioned LSCE 2007 book.

5. ANALYSIS OF STRUCTURES

Each elaborated theory, can be used for practical application by designers searching solutions in analytical or in numerical way. The analytical solutions, are very valuable, giving formulae ready for calculation of searched information. Unfortunately, most of such solutions concern of simplified tasks, only.

In some cases of bar structures were found analytical results, in other hybrid solutions (analytically- numerical) and numerical, only. The last seems to be useful for almost any type structures. In next chapters are given some information about elaborated by author approaches to analysis of different types structures.

5.1. Analytical solutions

Here can be shortly presented two domains of analytical solutions. The first for hexagonal plane grids and the second for separate, straight bars.

5.1.1. Solution for hexagonal grid

As it was shown in chapter 3.2, in some works were found analytical solutions for hexagonal bar infinite band grid (bar plate), loaded in each node by uniform loading (identical force in all nodes) or loaded regularly as in the Fig 26 a-d, by different boundary conditions on both edges.

The second solution concern of hexagonal grid freely supported on external circle and loaded by identical forces (perpendicular) in each node.

5.1.2. Examples of analytical solution for thin-walled bars

For single straight thin-walled bars were obtained following solutions, Refs 30, 35:

- Derived formulae for torsion angle and internal forces for 16 simple cases of loading and boundary conditions.
- Some examples of formulae for critical forces of types:
  - longitudinal force,
  - excentrical longitudinal force,
  - bending moments,
  - critical vibration of bar under action of longitudinal force.
  - Solutions for bar interacting with surrounding media: air, water, soil.

5.2. Numerical algorithms and solutions

There, were elaborated algorithms and programs for different types of structures: single straight bars, plane grids, wide class of space bar structures (including domes, cylindrical etc.), for elaboration of experimental results, calculations of geometrical characteristics of bar cross-sections, calculation of stresses etc. These algorithms were destined for: large computers with external memories, for PC computers, programmable calculators and for MS Excell.

5.2.1. Algorithms and solutions for some space bar structures

In the dissertation Ref 4 (Ph.D.) were presented some numerical tests for double layer small tasks, shown in the Figs 32-34 (plane and cylindrical). For this purposes were prepared some programs listed in chapter 6.2. They can be easily extended on next applications.

The other application of derived equations for hexagonal grids, were proposed in works Refs 4, 113, 135, 155. There, obtained analytically one equilibrium equation of node of finite differences character (for deflections, compare Fig 30) is applied to composition of stiffness matrix of whole structure. This way it is possible to have structures with more complicated shapes of its contour, and different loadings in each nodes. Such approach reduces drastically number of unknowns with regard to task, where are used three original equilibrium equations.

Next some more advanced algorithms, are shortly listed in next chapters.

5.2.2. Difference-Matrix Equation Method

This manner of composition (Refs 4, 11, 28, 29,135,) of stiffness matrix of structure was applied in two large programs KMT_D or KMT_G and in the most universal WDKM. There, were applied physical relations for single bar, shown in Table 5, structure geometrical description, as in chapter 4.3, analysis in range of static and dynamics, theory of first and second order, finite dimensions of nodes Figs 21, 54; any bar pattern, any support systems including inclined sliding joints, any boundary conditions for free displacement function of each bar (including rotations), any loading of large structures: in nodes and on the length of the bars (not to the end enoughly tested), possible declarations of any bar rigidities, etc.

5.2.3. Application of Finite Differences Method

The method firstly destined for teaching purposes, by means of universal program MRS, written by J.B. Obrębski (Refs 136, 155, 177; 17,52 kB, only) quickly was applied to scientific purposes, for static, stability and dynamics of straight bars (including bridges, tall buildings) and plates. The range of applications of the method, was extended on 3D-TSM for dynamics (see chapter 5.2.6 and Refs 133, 146, 165, 223, 232, 237). Wider description and examples of application were given in two fundamental works – Refs 30, 155 (there see for wider references).

The method gives possibility easily to take under consideration:

- different schemes of structure (including boundary conditions),
- variable rigidity of bar or platte,
- any kind of bar cross-sections (solid girders – homogenous or composite, tubes, rolled cross-sections, thin-walled, trusses, space bar trusses,
- variable –any loading system in each node,
- moving loads with variable path, velocity, jumping etc. (see chapter 5.2.6).

Simultaneously, the solutions, in spite of complicated character of tasks, is solved very easily. The definition of applied equilibrium equations, scheme of structure, its loading and support systems, bar rigidities –are declared from keyboard by user…

5.2.4. Own algorithms for space bar structures by Finite Element Method application

There, were built two fundamental programs

- small program FEM for didactic purposes in University of Warmia and Mazury in Olsztyn,
- large program system for technical optimization of mainly space bar domes for (written and tested with A.H. Fahema) named as SPES (SPace structurES).

Program system SPES consists of some programs for:

- printing scheme of structure,
- analysis of structure,
- analysis of calculated results (searching minimal or maximal values of displacements, internal forces, stresses, geometry of structure, volume of built in material, weight of structure, comparisons of some declared tasks (up to 20) etc.

5.2.5. Solutions for composite structures

Analysis of internal forces in elements of structures composed from composite bars (built of some materials forming longitudinal strips) can be done by means of all proper programs prepared by author, mentioned in this paper. Applying them, we shell introduce properly calculated bars rigidities, by means of theories given e.g. in books Refs 30, 32, 35 or by programs MB, MBK, MB-PC (see chapter 6.4)

5.2.6. 3D-Time Space Method for Dynamics

The general equations of motion in the case of dynamics and theory of second order are rather difficult to be applied to analytical solutions useful for technical real solutions. So, a closed analytical solutions for dynamical tasks are rather very simple of academic character. Such limitations disappear when FDM is applied even for combined loadings. Such solutions are easy to be executed even by means of commercial MS Excel program.

The *3D-Time Space Method* uses time as fourth dimension. There is applied ***F****inite* ***D****ifferences* ***M****ethod* and program MRS. This way, are available solutions of tasks, almost impossible to realization by other approaches. Here, very easily can be modeled: - impact single or multi loading, moving alone or group of loadings, the last – can have straight or curved in any way path, mass acceleration (each one separately), slacking, starting and stopping, changing direction of move, including opposite ones, etc. Moreover, loading can act with different intensity and/or velocity, or jumping (landing aircraft) etc. The same can concern of contact problems for supports, etc. So, program of loading can be applied as variable in 3D space and in time...

In the method, accordingly to scheme given in the Fig 60, behaviour of the structure in each time moment t considered individually, is included to common task as it is explained in the Figs 61, 62 (J.B.Obrębski and Szmit Ref 223).

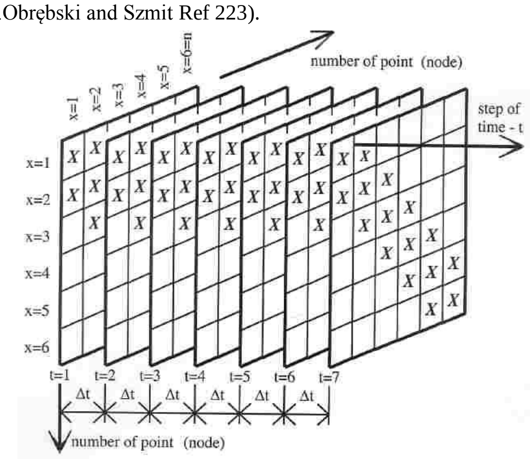


Fig. 60. Real scheme of 3D-Time task numerical analysis.

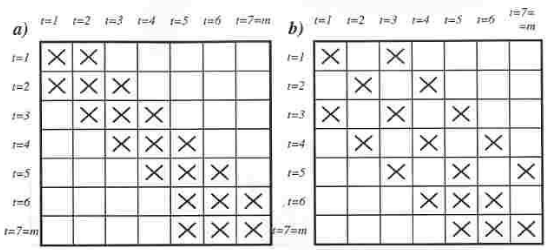


Fig. 61. Numerical representations of the 3D-T space of the Fig.60.

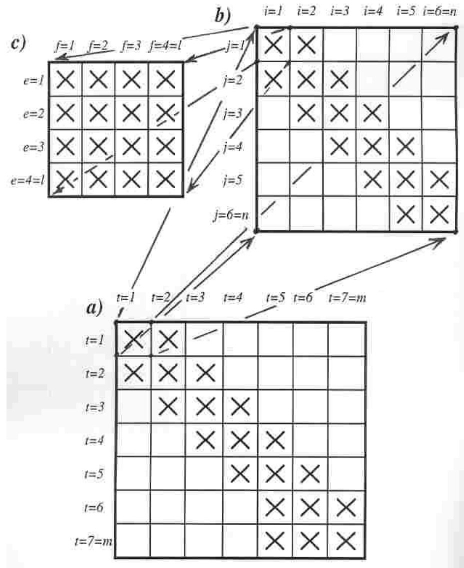


Fig. 62. Detailed scheme of 2D dynamical stiffness matrix for structure considered in 3D-TSM with 6 nodes (b), 4 degrees of freedom per node (c) during 7 time moments (a).

Some such examples were presented in previous author's works. There, as central point of numerical algorithm is solution of Eqn (9), where this time matrix K is called as *dynamical stiffness matrix*. Applying FDM we can modeling many tasks, steering proper steps along all of four axes of 3D-T space. As particular cases, there can be used 2D-T (plates, shells) or 1D-T (beams) spaces. From numerical point of view, always it is 2D problem – two dimensional, square stiffness matrix K Eqns 21-23, Refs 115, 122, 124, 128, 133, 149, 169, 223, 232, 238, 240, 256.

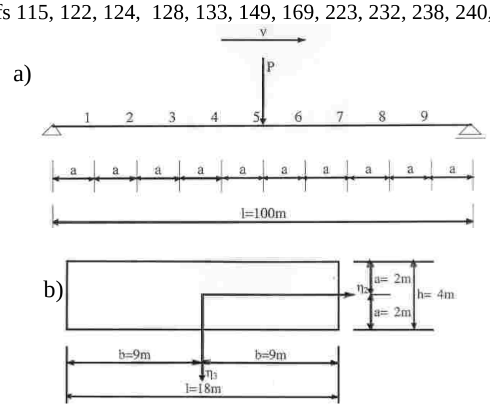


Fig. 63. Scheme of bridge with moving load a), with bridge cross-section b).

Proposed approach is very similar to DMEM, now applied to *3D-Time space* (3D-T). It brings us to name of the: 3D-Time Space Difference-Matrix Equation Method (3DT-DMEM). All kinds of above approaches, together will be named 3D-TSM.

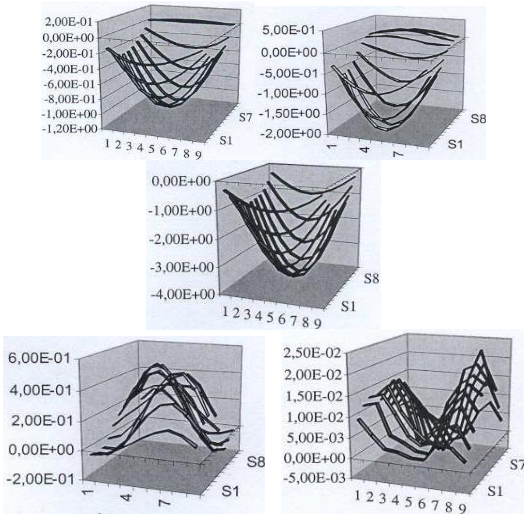


Fig. 64. Diagrams of bridge deflections [cm] under moving load on bridge with span 100m (Fig.63) in eleven time moments (series), with parameters: loading mass G=100t, loading velocity v=36, 180, 360, 720, 3600km/h, bridge mass μ=0.022 695 016kNs^2/cm, its rigidity EI=2.083 333E+13 kNcm^2, Refs 240, 241.

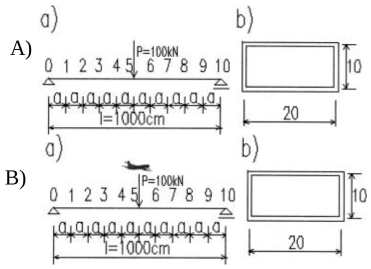


Fig.65. Freely supported steel box beam with moving load Ref 238; v=36km/h A) v=360 km/h B).

Simplicity of the method, allow on its implementation by R.Szmit for tall-buildings, on standard PC computer series Pentium, using MathCAD and Excel programs, too (Ph.D. dissertation Ref 281). Mentioned commercial programs as to general, were not comfortable and not to the end efficient for 3DT-DMEM application.

Therefore, in the J.B.Obrębski's works Refs 105, 232, 235-238 the attention was turned on numerical implementation of the method, where is possible:

- to apply special, general and simple program MRS (very small 17,52kB!!) author J.B.Obrębski (Refs 110, 113, 117, 135, 155, 240), used for teaching of the mechanics principles for beams and plates etc.),
- to use any kind of sets of equilibrium equations, of ***F****inite* ***D****ifferences* (first or second order etc.), including tasks concerning of 3D-Time space, where the method is oriented on straight bars and in it: tall-buildings, bridges, foundation piles (driving in) etc.
- application of above simple standard program for the beams, where is possible to consider influence of elastic three-parametrical Winkler foundation, interaction with wind or fluid, friction etc., to plates and shells including dynamics and stability, too (J.B. Obrębski Ref 30),
- modelling of above structures, homogenous, anisotropic and composite,
- modelling of movable loadings – e.g. car(s) on a bridge (as beam or as plate),
- to use advantages following of repeatability of the structure nodes and loading,
- to consider simple-, elastic-, rigid and intermediate supports of investigated structures.

The works of J.B.Obrębski Refs 237, 238, 240, 241 over the 3D-TSM gives positive answers on all above questions bring us to next category, exact and relatively simple numerical engineering solutions. They are easy in application and comparative or even often better then FEM results.

The numerical examples shows Figs.63-65. In the last example the velocity (about 720km/h) of moving mass G can be considered as critical one, Figs. 63, 64.

6. OWN PROGRAMS AND SYSTEMS FOR ANALYSIS AND SYNTHESIS OF STRUCTURES

Theories associated with these problems, are rather complicated - mathematically advanced and therefore laborious by practical calculations. So, for such easy reason all above problems were solved by means of computer. The particular problems, accordingly to development of computer technology, were implemented on successive, the most popular in Poland computers, of particular its generations, Ref 85, 94, 117, 155.

The programs are based on own theories named: ***D****ifference-****M****atrix* ***E****quation* ***M****ethod*, on ***F****inite* ***E****lement* ***M****ethod* (FEM) and even of ***F****inite* ***D****ifferences* ***M****ethod* – including ***3D-T****ime* ***S****pace* ***M****ethod* (for dynamics) dependently on its destination and analysed problems. Moreover, some programs are oriented on strength analysis of single bars including these with thin-walled cross-sections, full and composite ones. There are applied algorithms using many theoretical formulae.

The author from 1970 was writing with different intensity the computer programs and systems oriented on needs of civil engineering. In this time period, programs were written in many languages, from which to more important belong: *Odra Algol, Algol 1204, Fortran, Turbo Pascal i C^{++}*. In years 1974-1999, specially intensively was working on large program systems, documented in book form, written in Polish and German Refs 28, 29. Numerous papers were published in conference proceedings in English, too. These programs can be assembled in five groups.

6.1. Programs for solution of sets of linear algebraic equations.

There, should be mentioned:

- solution of set of algebraic linear equations by Gauss method for symmetrical matrix, Fig 67,
- solution of set of algebraic linear equations by Gauss method for general square matrix, Fig 66.

Above programs were written in many versions:

- executing calculations in computer memory, only,
- using computer memory and external memories (drums and magnetic tapes),
- using computer memory and virtual drums, etc.

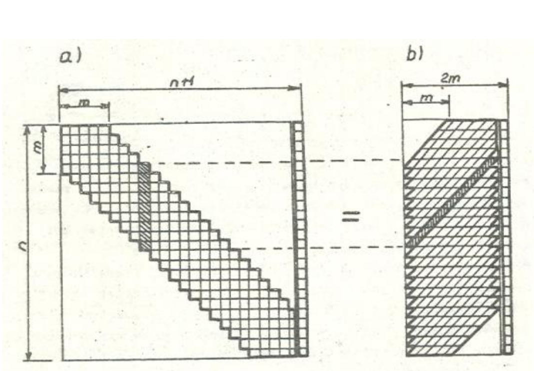


Fig. 66. Scheme of unsymmetrical set of linear algebraic equations.

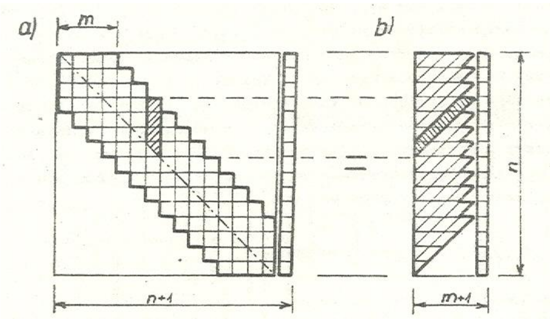


Fig. 67. Scheme of symmetrical set of algebraic equations.

6.2. Elaborated own programs oriented on hexagonal structures
Programs in Algol for:

- circular plane hexagonal grid,
- double-layer plane space bar truss type I (Fig , Ref 4),
- double-layer plane space bar truss type II (Fig , Refs 3,4),
- cylindrical double-layer space bar truss (type II Fig ,Refs 3,4),

6.3. Programs for large space bar structures
Here, can be pointed some following programs:
KM – program written in Algol 1204, latter translated on FORTRAN. It is destined for analysis of structures with nodes inscribed in orthogonal net of points, only.
KMT$_D$ – larger version of *KM* program, on *Odra 1305* computer, written in FORTRAN, extended on automatic dimensioning of circular, tubular cross-sections, accordingly to allowable stresses method, accordingly to PN-64/B-03220 (aluminium structures) or PN-76/B-03200 (steel structures). There, program proposes proper size of bars cross-sections, by defined structure global geometry, types of its cross-sections, loading system and supporting system.
KMT$_G$ – identical program as *KMT$_D$* , but automatically dimensioning of circular straight tubes, accordingly to ultimate states method, by PN-80/B-03200.

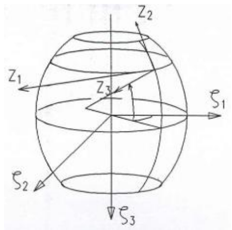


Fig. 68. Definition of simplest, double curvature, revolution net of points.

WDKM – it is system of about 314 cooperating procedures written in FORTRAN for *Odra 1305* computer. It enables analysis and automatic dimensioning of structures inscribed in four types of net of points Refs 11, 14, 15 etc.:

- orthogonal, Figs 44, 45,
- rotational with two centres of curvatures, Fig 51a, 68,
- translational net of points determined by any type mathematical surface (translated along axis z) and two families of vertical planes, Fig 50a,b, 69.
- revolution-translational nets of points with leading, mathematical line rotating around axis, Fig 50c,d.

Global shape of structure can be changed in input data by declaration other number of a few parameters.

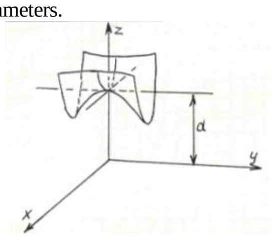


Fig. 69. Definition of the translational nets of points - here as leading global surface is used hyperbolical paraboloid and $d=z_o+\Delta z$.

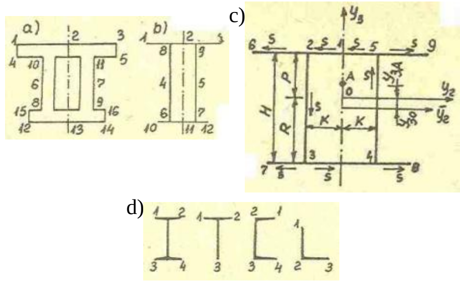


Fig. 70 Types of cross-sections automaticly dimensioned in WDKM program system, with shown points for calculation of stresses (compre Fig. 22).

So described the structures global geometry permits in easy manner to generate descriptions and input data for specially large space bar structures, one- and two-layer: spherical, cylindrical vaults, barrel, conical, toroidal etc. This way can be easily described and analysed similar structures as generated by Formian (by H.Nooshin) and tensegrity domes proposed by J.Rębielak.

SPES – it is system of a few cooperating programs, composed of about 100 common procedures. It is destined for analysis and semi-optimization of rotational, elliptical, complicated space bar domes: single- and multi-layered. It was built in cooperation with A.H.Fahema (Libya) for his doctor thesis. Three essential programs for input data, solver and analyser were written in FORTRAN and two in C^{++} for PC computers. These programs can describe, analyse and optimize trusses or frames with general shape identical as by *WDKM*, extended on elliptic, and cyclic wavy domes, with vertical and/or horizontal waves.

There, is possible static analysis, only, and comparisons up to 20 similar structures, with regard to up to 20 objective criterions of optimization. Some diagrams facilitate to compare results of investigated objective criterions. It permits for designer to do last decision about the structure choice. There are possible to be compared: maximal forces, stresses, displacements, elastic work of structure, elevation of highest node, number of nodes, bars etc. Description of structures is there similar and probably sometime wider as in H.Nooshin's FORMIAN programs.

6.4. Programs for bars strength analysis
It were built some small programs for auxiliary tasks of strength calculations for straight bars with any cross-sections. They were written in Turbo Pascal:
MB – program for calculation of geometrical characteristics of the bars: area, gravity centre, position of principal axes and inertia moments. The cross-section is modelled from smaller elementary fugures as: rectangular, triangle, one quarter of circle and circle.
MBK – Program for calculation of geometrical characteristics of composite cross-sections. It is similar to *MB* program, but there each elementary figure, defining whole cross-section, can be declared as made of different materials [11].
MB-PC – it is the program in Turbo Pascal, producing similar results as *MB* or *MBK* programs, but for thin-walled rectangular cross-sections open or closed. There is graphics for drawing diagrams of: coordinates, sectorial coordinates, usual- and sectorial -statical moments.
NG – program for calculation of principal stresses and its directions, for plane and 3D states of stresses.
NK – program for calculation of critical forces and stresses, too, for straight bar.
HMH – program calculating values of reduced stresses accordingly to *Huber-Mizes-Henckey* hypothesis. It draws for given material ultimate curve and shows position of actual state of stresses.
STAN – this program calculate and draws values and diagrams of internal forces, geometrical characteristics, including warping ones and stresses for cantilever thin-walled beam with open or closed rectangular cross-section and by two cases of boundary conditions, Fig 4A. Identical beams were investigated experimentally for checking foundations of theories for thin-walled bars Refs 30, 32, 35.

6.5. Programs to elaboration of own experimental results.
Here, are listed small programs, destined for facilitating elaboration of experimental results for beams analysed by program STAN, too. They are written in Turbo Pascal.
UI – this program elaborates longitudinal and circuital displacements in particular cross-section.
U3 – the program elaborates displacements perpendicular to bar cross-section, for the same cantilever beam.
TEL – elaborates experimental results of electro-resistance measurements for considered bar. It calculates strains, stresses and draw proper diagrams.
TELW – it draws the diagrams, only, on basis of data produced by program *TEL*.
TAB – this program calculates values of such internal forces as bimoment and bending-torsion moment using experimental results. It is done for bars identical as calculated by programs *STAN*, *MB-PC*, *TEL* and *TELW*.
YOUNG – it calculates Young's modulus measured experimentally.
BETA – it enables calculation of certain coefficient correcting bar torsional rigidity for thin-walled, investigated as above bars (see theories of Vlasov and Ref 30, 32).

6.6. Programs Elaborated Specially for Didactic.
There were written two programs MES and MRS destined in the beginning for students teaching in University of Warmia and Mazury and in Warsaw University of Technology, on faculties of Civil Engineering, both Poland. The programs are written in Turbo Pascal and have no any graphic. Its destination is to show for students principles of foundations relatively ***F**inite **E**lement **M**ethod* and ***F**inite **D**ifferences **M**ethod*.
MES – the program enables analysis of small plane trusses and frames, producing displacements, internal forces and reactions. It has the didactic destination, only.
MRS – the program is oriented on analysis by Finite Differences Method of any task which can be described by differential equations, transformed to finite differences operators with defined boundary conditions. These operators – equilibrium equations and next physical relations for internal forces, are defined in input data... So, the program is very universal. Till now, it was used for: bended bars and plates, for bars and plates on elastic foundation, for stability problems, for 3D-Time Space Method for dynamics of tall buildings and bridges under moving loads, etc. There, the analysed bars can have full or thin-walled cross-sections, to be homogenous or composite, to have constant or variable cross-sections on its length etc. There can be applied any loading system – variable in 3D and in time, too.

6.7. Numerical dimensioning of bar structures
To special function of programs KMT_D , KMT_G and WDKM belong automatic dimensioning of bar structures. The programs for given structure, with defined scheme (bar pattern, support system, loading system, declared types of bar cross-sections) are searching dimensions of cross-sections, assuring safe state of stresses in whole object. The procedure of selection of proper dimensions has iterative character and run in maximum 3 steps.

6.8. Optimisation and semi-optimization of large space bar domes
Below are given some drawings (Ref 17, Figs 71-76), only, showing possibilities of the programs in two domains – analysis and shape and form finding. The wider comments and next examples can be shown during presentation.

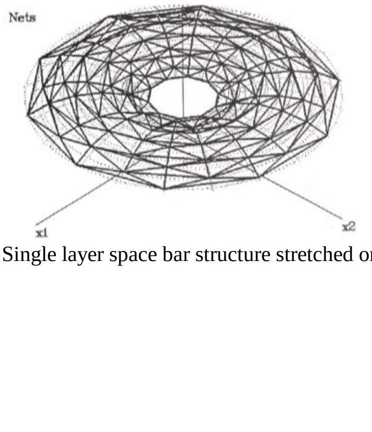


Fig. 71. Single layer space bar structure stretched on torus.

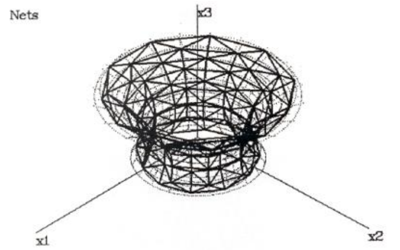


Fig. 72. Single layer space bar structure stretched on torus.

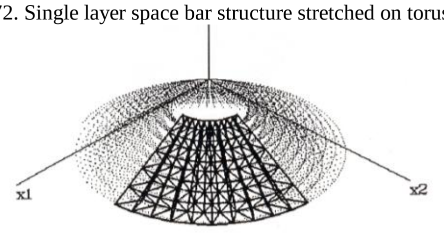


Fig. 73. Scheme of double layer, conical space bar structure.

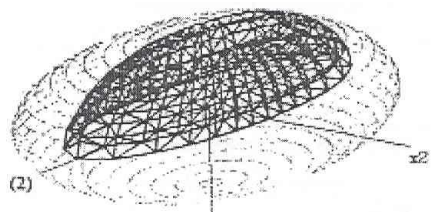


Fig. 74. Scheme of double layer, elliptical space bar structure.

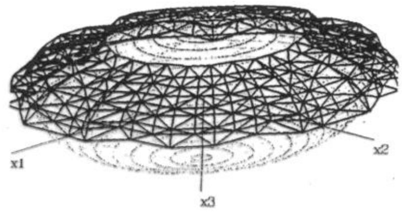


Fig. 75. Scheme of double layer, wavy space bar structure – horizontal waves.

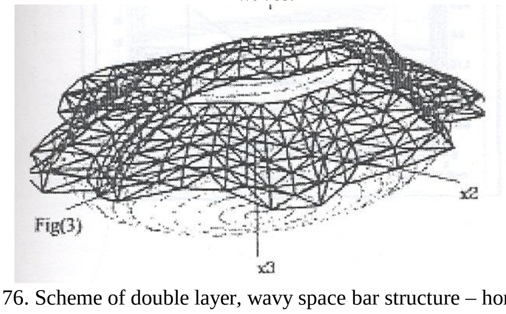


Fig. 76. Scheme of double layer, wavy space bar structure – horizontal and vertical waves.

7. OBSERVED EXACTNESS OF STRUCTURES ANALYSIS
In literature are accessible some reported information, about errors, which can be made by incapable or improper application of analyses theoretical or computer, too. The author was publishing some papers discussing such problems, Refs 77, 139, 142, 160, 162, 163, 167, 172, 234, 247.

There, are well known examples (below are quoted a few, only) that:
- application FEM to analysis of single bars can give error up to 394%, Pruki and Lopez Ref 302 (2001); - by analysis of core for tall building, warping normal stresses – from torsion, can reach 270% of normal stresses from bending, Smith and Coull, Ref 304 (1991);
- taking not into consideration of reinforcement in bended concrete elements, gives deflections bigger e.g. about 95% (numerical tests, Obrębski (LSCE 1995-2006);
- torsion in most of computer analyses is to simply described, Obrębski (LSCE 1995-2006),

- critical compressing force of bending-torsion type for steel column (high 400cm, with I 20x20 CS), reach value P_{cr}=1202kN, when critical force of bending type only (Euler), has value P_{cr}=1852kN, (omitting torsion we obtain error about 54%), book Obrębski Ref 30 (1991).

It is well known, that for any type bars (**TW**, compact, homogenous or composite) **with torsion appears bimoment and bending-torsion moment**, both generating significant warping stresses.

7.1. Accuracy of designing process in the light of contemporary knowledge.

In some author's papers were presented observations on some methods of structures analyses, which can make designing process of much higher quality and erected objects much more safe. It concern of some steps of designing process, mainly concerning of structure analysis: stiffness calculation, determination of internal forces, stresses calculations and at last dimensioning of cross-sections (**CS**s). On each of these steps, by nowadays widely applied approaches, can be generated significant errors. Such conclusion follows of compared some results of given similar analyses: analytical, numerical and experimental, performed for many different objects and types of investigated tasks, own and quoted in literature. There are pointed high uncertainty obtained results, especially often when produced by computer. Next, in some other analytical and experimental examples, is visible strong influence of bimoment on stresses and on instability of thin-walled bars. The other serious problem concern of the mechanics of structures built of composite bars. So, it is recommended to apply better theories and to prove and evaluate obtained results in some independent ways, including experiments, too.

7.2. Short Description of Quoted Examples - Results and Comments

□ **Examples Known From Literature.** In the papers Refs 172, 247, were shown some extreme examples which indicate possible errors.

► FEM Tests. Pruki & Lopes Ref 303, an example of freely supported concrete beam, uniformly loaded, have given. There, were compared longitudinal stresses calculated by three well known programs and errors determined by uniform formula. There were presented results for 53 types of beam divisions, and different type finite elements, totally 172 times! The error **e**<1% with comparison to analytical solution was obtained 7 times, only… (4.06% of all examples); **e**>20% in 91 tests (52.9%); **e**>50% in 33 cases (19.18%); **e**>100% to 394%, in 10 cases (5.81%).

► Experiments and FEM Results. Glinicka in she's habilitation thesis, Ref 297, hes investigated thin-walled steel girders for window or door headers, loaded by two concentrated forces. The steel beams have rectangular closed box cross-sections or of some open types, filled by foamed concrete, too. Comparing values of loadings giving deflection value 3 mm, in one case only convergence of experimental and numerical results was quite well, in remaining errors were from e=15.6% up to 88.8%.

► Torsion of thin-walled bars. There, coefficient β correcting calculated bar torsion rigidity, measured by Obrębski and Urbaniak (Refs 195, 197) was verified by Jankowska using FEM, Ref 119. The idea was simple. The thin-walled bar is loaded by torsion moment and proper torsion angle must be calculated or measured, and next both were used for calculations. The experimental curves are strongly nonlinear when adequate numerical ones are almost linear. Moreover, three curves for the same bar, dependently on applied elements, are almost horizontal lines with values β=1.5; 2.55; 2.75 (there **p**=2.75/1.5**=1.83).**

► Comparison of Optical and FEM Approaches. Dymny et al were presented investigations performed on order of European Union. There, special plane specimen was tested. Interferograms obtained experimentally and synthetic calculated numerically, were compared. Differences of two pictures obtained in both above ways are very strong.

► Symmetry by Numerical Solutions. In the master degree theses were investigated double layer space bar structures. There, as purpose was checking the exactness of numerical calculations. Symmetry of structure was modelled in three ways. Nearby plane of symmetry internal forces were smaller up to 21.4% and deflections for quarter of structure bigger up to +14.2% of obtained for whole structure. Part of the structure is working as having smaller rigidity Refs 263, 264.

► Other Experiments (LSCE 2002). The first by Gleich, concern verification of numerical analysis of adhesive connections. By oral presentation was reported dramatic difference of diagrams character of failure load - theoretical and experimental. The example by Meier et al concern of strengthening of reinforced bridges and tubular masts by carbon-fibre reinforced polymers. Curves obtained experimentally and calculated for ultimate load are remarkably different.

□ **Composite Bars.** For such bars is destined theory Refs 30, 35 (see LSCE 95, 2004, too). There, if we change value of general Young's modulus, normal stresses, bar elongation and strains are still the same, but values of reduced characteristics are changed. Moreover, the composite bars, in all cases have the rigidities much higher, than known from traditional strength of materials. It has influence on stresses and displacements. So, numerical analyses should take into consideration reduced geometrical characteristics, on stages of internal forces and stresses calculation.

□ **Some Numerical Tests** on Simple and Space Bar Structures with variable **CS**s.

►Cantilever beam with constant and variable rigidity, triangle or trapezium type (LSCE 2002). Were derived analytically formulae and values of deflections. These results were compared with numerical calculations done by Finite Differences Method, with differential equation of fourth order. At the end the triangular cantilever, rigidity $EI_2=0$ and the analysis is impossible (division by zero). In the case of trapezium type bar by FDM variable rigidity is not visible for algorithm.

► Frames With Variable Rigidity. The test concern of three plane frames having variable rigidity of bars (more stiff ends). All bars have thin-walled rectangular cross-section 10x20cm of two types – open and closed with two walls thicknesses. The numerical algorithms needs to calculate torsion moments of inertia I_1 (Refs 30, 32). It depends on the length of bar section and on functions sh(x) and ch(x). There, computers accepts x<224, only. It brings on computer approaches strong limitations.

□ **Next Observations and Proposed Theories.**

►In next examples it is visible influence on results quality of applied: methods, theoretical or numerical model and input data for numerical analysis.

► The bimoment is evidently real internal force, very dangerous for structures, which should be seriously considered by designing of objects composed specially of thin-walled bars. In nowadays analyses, computer programs and standards, the bimoment is completely ignored! Instead of possible good analytical analysis (Refs 30,35) its influence is taken into consideration applying some empirical coefficients, assumption of "effective" (?) cross-sections etc.

►Numerical modelling of shape finding gives certain approximation of results obtained experimentally (Ramm's shape numerical optimisation, and Isler's experiment)

►Applying author's, new theories and programs - for strength analysis of composite bars, for global analysis of space bar structures (Refs 11, 30, 35) and the 3D-Time Space Method for dynamics of some type tasks, etc. the possibility to do big errors is seriously reduced.

7.3. Evaluation of computer measurements by modern System 5000 of the firm VISHAY.

There (Ref. 282), were applied rosettes with basis of each of the three sensors 3 mm. In spite of careful preparation of measurements, the 4 sensors in 3 cross-sections (on 32) were not working.

It seems, that by such excellent equipment should to give very exact results. Unfortunately, after careful examination of presented materials (Ref. 282) we find, that there we can have many to wish. So, in 3 cross-sections of model type L were applied 8 loading levels. In proper tables for stresses we find lack of results (inscription - *#ARG!*) for all 25 measured points for first loading level and minimum one such inscription in first 3 to 6 loading levels, on 8! Similarly, in remaining 29 cross-sections of frames type L, T and Y, were applied 9 loading levels. There also was lack of results (inscription - *#ARG!*) for all 25 measured points of first loading level and minimum one such inscription for 3 to 9 loading levels (on 9!!). Even in 4 cross-sections on 32 were not presented error-less results. So, in 4 cross-sections on 32 were not presented complete results.

In 3 next cross-sections, full information was given for the highest (9th) loading level.

So, it can be concluded, that by computer elaboration of electro-resistance measurements results, the quality of obtained information was rather weak. Contrary, by manual measurements and results elaboration, similar problems were not observed.

7.4. Summary to the problems of accuracy of structures analysis

■ Computer results in many cases can be regarded as certain simulation of the phenomena, only. It often gives the approximate evaluation of investigated problem.

■ Preparation of complicated projects, without experimental verification is unbelievable. The part of experiment in evaluation of structure behaviour is rather without any discussion.

■ Obtained errors in numerical analyses applying FEM can reach even 394%. So, we shell be very cautious with application of its results.

■ Application of reduced geometrical characteristics on each stages of bar (and other) structures analyses, is recommended. It permits to eliminate significant errors in evaluation of internal forces, displacements and stresses.

■ Still, in some situations, even applying computers we have serious limitation of exactness of obtained results.

■ Calculation of the half or the quarter of symmetrical structures, can give remarkable errors!

■ Some of calculated results are dramatically different from reality. A progress can be obtained after its verification or calibration by experiments.

Next conclusions the author leaves for the reader. In quoted references are quoted next wider literature lists. Quoted papers gives more information about some possible sources of errors.

8. SUPERVISED WORKS

The author has promoted some dissertations, as well of M.Sc. level as Ph.D. , too. Below is given its short description.

8.1. Supervised master degree works

All together there were prepared and successfully finished 17 such works. They can be classified in following manner:

- analysis of thin-walled bars, Refs 259, 267, 271,
- experimental analysis of thin-walled bars, Ref 272,
- analysis of space bar structures – plane roofs, Refs 262, 263, 264,
- buildings, Refs 260, 261, 265,
- bridges 266,268,
- numerical programs, Refs 269, 270,
- projects, Refs 273-275.

Below are shown some pictures explaining categories of investigated structures.

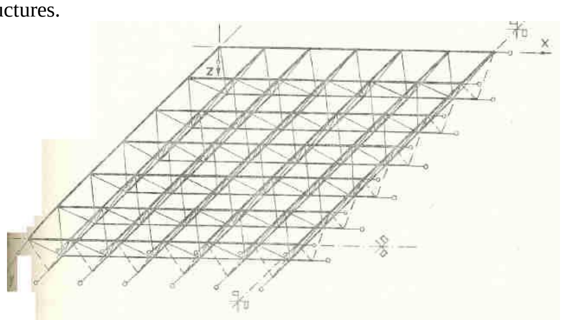


Fig. 77. Double layer truss investigated by M.Kowalski Ref 264. LSCE 2005 ¼: Nodes – 97-, unknowns – 291-294, members – 324-328.

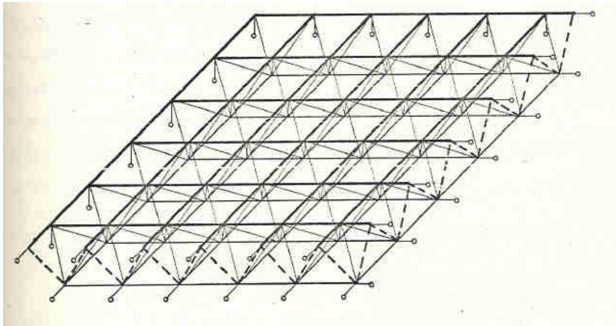


Fig. 78. W. Słabosz, Ref 263, LSCE 2005. Nodes – 84-265, unknowns up to – 795,

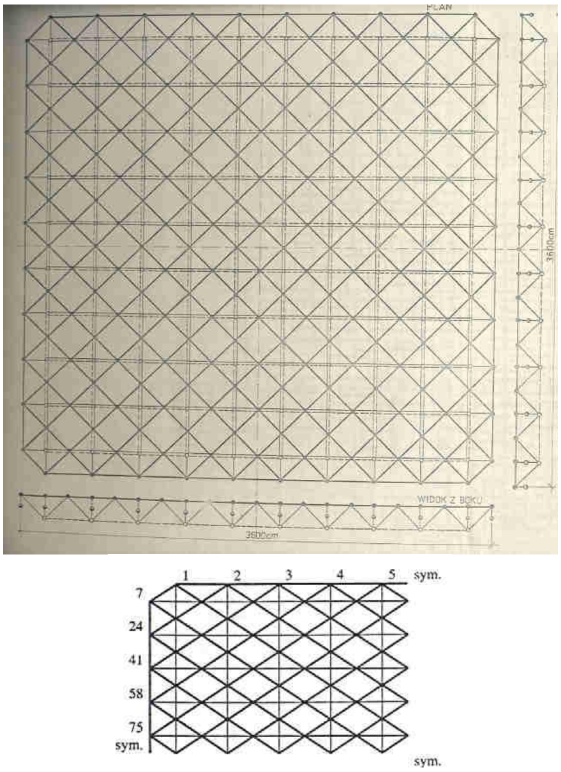


Fig. 79. Double layer truss investigated by B. Zbyszyński; nodes – 97-630, unknowns up to – 1890, Ref 262.

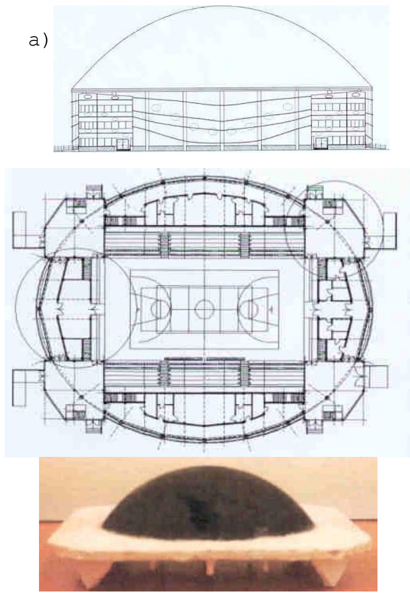


Fig. 80. Diploma project, by P.Kierzkowski and J.Tolksdorf, Refs 274, 275.

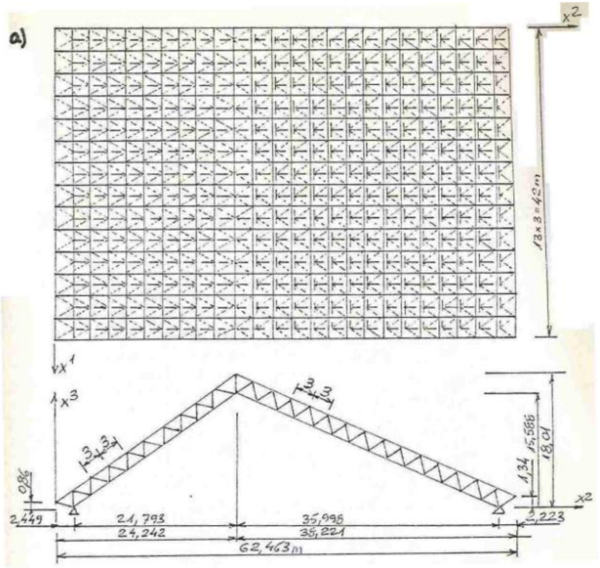


Fig. 81. Diploma project, supervisor J. Karczewski, analysis by progtam KMT_D. Nodes – 697, unknowns – 2091, wide of half band - 92

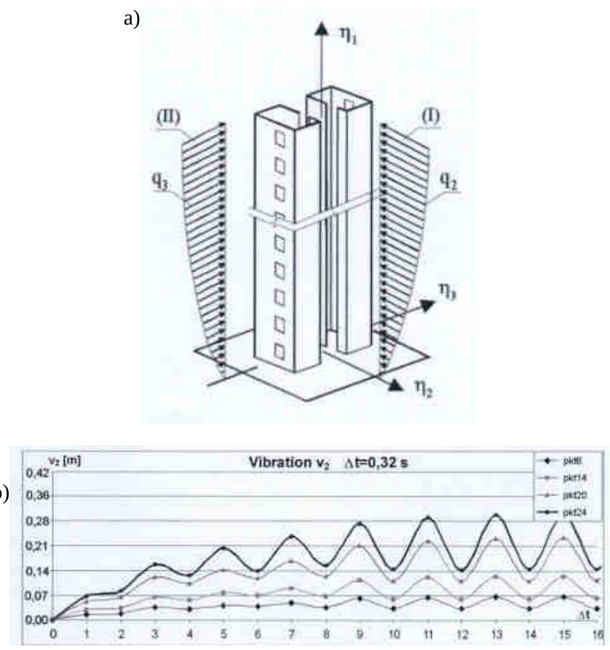
8.2. Supervised Philosophy Doctor dissertations

There, were successfully elaborated and finished 6 such dissertations. Two next are continued. These Ph.D. dissertations can be classified in following manner:

-analysis of thin-walled bars, Refs 276, 277, 278, 283,
- space bar domes, Refs 279, 280,
- tall buildings, Ref 281,
- analysis of thin-walled frames, Ref 282,

Some examples of structures investigated in this category dissertations are presented by following Figures.

a)



b)

Fig. 82. a) scheme and b) defections of tall building, investigated by R.Szmit, Ref 281.

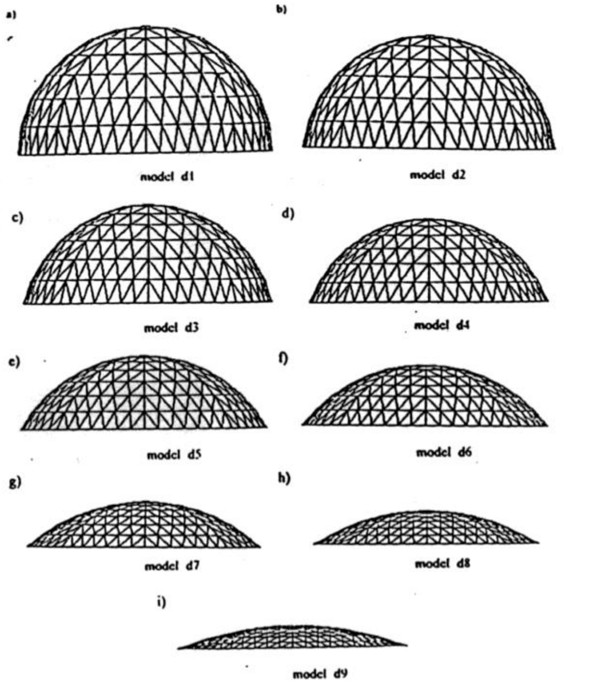


Fig. 83. Variants of dome investigated by A.H.Rhuma, Ref 280, for optimal solution. Nodes – 225, unknowns – 675, members – 616, span 30m.

9. PARTICIPATION IN DESIGNED PROJECTS

Below are presented a few examples of structures calculated by programs elaborated by author. The general schemes and some numerical parameters are quoted, only.

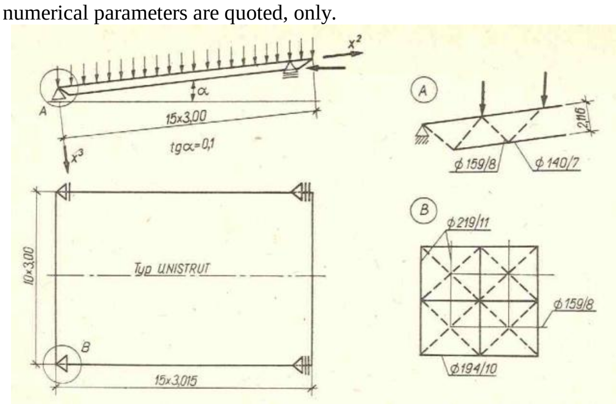


Fig. 84. Scheme of calculated shipyard hall in Szczecin, (app. 1978) Nodes – 326, unknowns – 978, wide of half-band – 66, Ref 28, 29.

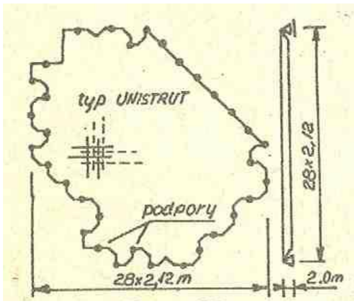


Fig. 85. Scheme of MEGASAM - market hall in Warsaw, app.1978. Nodes – 1079, unknowns – 3237, wide of half band – 165, Ref 11.

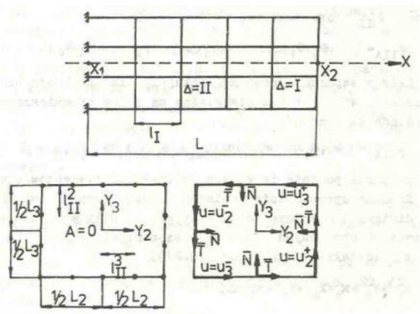


Fig. 86. Scheme of frame investigated by R. Nagórski, Ref 299.

The other example, calculated for purposes of habilitation thesis of R. Nagórski, concern of a frame of tall building, with rectangular cross-section. The bars of frame form the tubular frame with 312 nodes (24 on each story – net 4 x 8 modulae, 13 levels), including 24 supporting points wholly fixed or sliding articulated joint (hinge. The task has 1972 degrees of freedom, with width of half-band – 150. Time of computations was 4.5 hours. The calculations were performed for comparison with own theory of R. Nagórski with the others, Ref 299.

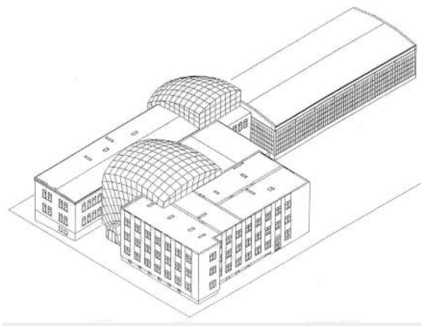


Fig. 87. Competition project – bird view of proposed two torus type ”gaps” to be infilled to the building of Faculty of Technical Sciences on the campus of Warmia and Mazury University in Olsztyn, Refs 145, 162.

10. WRITTEN BOOKS AND OTHER ACTIVITY

The author has written alone 4 monographs and 2 as co-author. Here in a few photos are presented its front covers Figs 89-92.

10.1. Recommendations.

Separate domain of author's activity and publications, besides of above shown books, constitute popularization of new ideas and technologies:

- known lightweight structures,
- evaluation of known structural systems,
- new structural systems, materials, theories, method of analysis and synthesis.

To this category of papers belongs e.g. Refs 117,123, 142, 148, 162, 167, 177, 229, 242, 246, 248, 251, 253 etc.

11. CONCLUSIONS

These all own elaborated theories, algorithms and programs permit nowadays, together, to design much better then in the past. But the man – user and designer, is there still in central point of each technical project.

Short review of mentioned above problems permit on formulation of the following principal conclusions.

1. In many tasks it is possibility to analyze the structures much more exactly, applying better, proposed here uniform theory.
2. The critical state of loading can be calculated for any set of combined loadings. It is possible to find there the critical curves and even critical surfaces. It is possible for composite bars, too Refs 225, 227, 170.
3. Application of Finite Differences Method seriously is extending a range of possible solutions.
4. It is now evident, that nowadays methods of analysis should be revised and completed by some elements of presented above theory. It should significantly improve safety of designed structures.
5. There, are recommended theories and programs elaborated by author.
6. This large paper can be, after improving and extending, certain skeleton of a monograph about possible modern analysis of mainly bar structures.
7. In this paper, the more important remarks and conclusions are presented, only.
8. At last, it should be pointed, that nowadays, in standards torsion and bimoment are almost completely neglected.
9. Moreover, there is big tendency to eliminate from dimensioning process of stresses calculations. **It all together, appears as highly dangerous**.
10. Besides experiences with mentioned above performed own programs, effectively were applied some commercial programs as ROBOT, Math-CAD, EXCELL, etc. which can be used for complicated calculations and results presentation of many tasks.

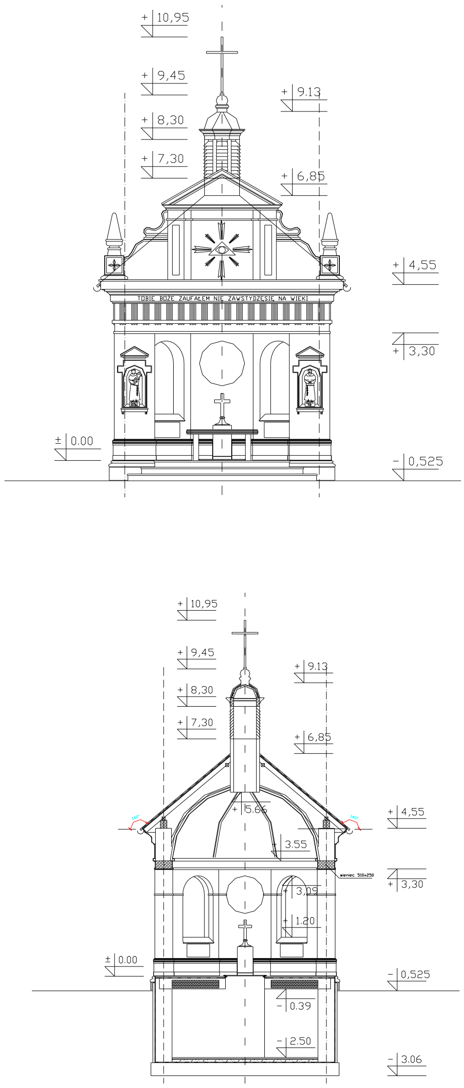


Fig. 88. Cementary chappel, front view and cross-section (designers: J.B. Obrębski – conception and construction; Konrad Obrębski – architecture), Ref 166, 167.

Fig. 89. Doctor and Habilitation thesis of the author, Refs 4, 11

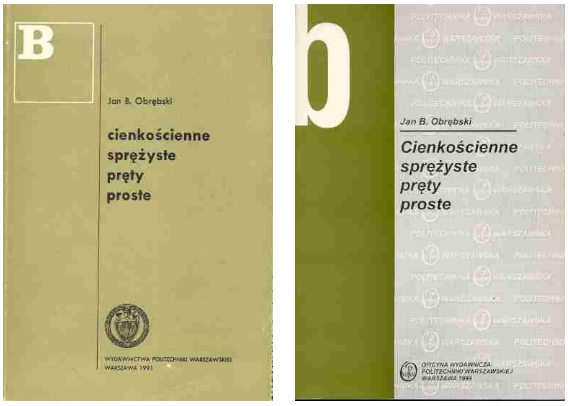


Fig. 90. Two editions of lecture notes, Refs 30, 32

Fig. 91. Two editions of the book on static of structural bar roofs, Refs 28, 29

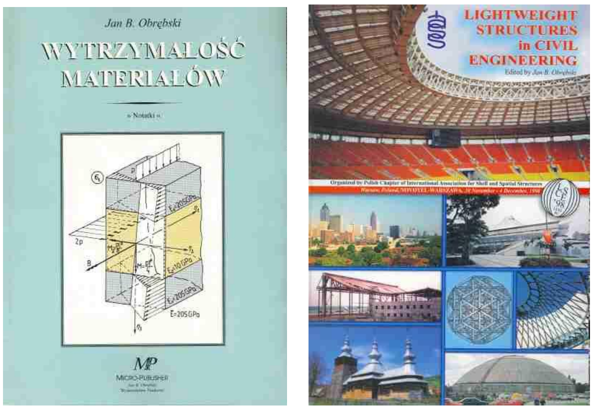


Fig. 92. Lecture notes – “Strength of Materials” Ref 35, and example of 4th proceedings of the International Colloquium on Lightweight Structures in Civil Engineering, Warsaw, 1998, Ref 36.

12. REFERENCES

Attention: in the listed below works, books and paper are omitted extensive volumes of 9 manuals for WDKM computer system, ordered works, academic reports etc.

Original published creative papers

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283. Jan Tolksdorf: Analza wytrzymałości i stateczności sprężystych prętów prostych pod obciążeniem złożonym. (Analyzis of strength and stablity straight bars dunder combined loading. Open on 06.06.2007.

Scientific-research works and project applied practically

284. Program on digital machines ODRA 1204/1304/1305/CYBER 70, (1974/75). Oryginal system of input data and numerical algorithm for program KMT used next in other authors programs. Program several times was used for designing calculations for designing offices. Part of author in program elaboration 70%.
285. Large program system (1976-80) WDKM for space bar structures cooperating with coninual elements; static, dynamics, stability, numerical dimensioning. ODRA s.1305. Many practical analysis of space structures, including for MOSTOSTAL directly or as M.Sc diplomas. Own part 85%.
286. Program system WDKM, (continuation 1981-85). Several scientific calculations e.g. for Ref 300 by habliltation prof. R. Nagórski. Lectures for students of Theory of Structures (Teoria Konstrukcji). Part of author 95%.
287. Theory of thin-walled bars supported by experiments (1986-90); with any cross-sections - static, dynamics, statbility, interactions with surrounding media (soil, air, water), dynamical stability. Teory 100%, experiments and elaboration of results - 80%.
288. Implementation of elaboratem theories to lectures on two faculties in Warsaw University of Technology: Strength of Materials (ISIW PW (89-91). - IL PW); Mechanics of Thin-Walled Bars in Warsaw and in Olsztyn.
289. Elaboration of didactic programs MES, MRS and others in Turbo Pascal. Lectures and classes for computer Methods in Mechanics in ART Olsztyn (later UWM Olsztyn).
290. Theory for composite bars with any cross-sections. Implemented to lectures and classes in Strength of Materiale (II, IL PW, Refs 30, 32, 35). 100%.
291. About 30 „invited lectures" around the world on: - mechanics of composite and thin-walled bars; torsion of bar structures; mechanics of space bar structures; morphology of selected space bar structures and Domes since 1995. 100%.
292. Elaboration of algorithms and programs for practical, limited optimization of complicated, large space bar structures (system SPES bazed on FEM. 70%.
293. Elaboration of new methods of application of Finite Differences to analysis of structures including dynamics (3D-TSM), by program MRS. 80%.
294. Experimental investigations of thin-walled bars in range of influence of torsion; Refs 277, 282. 50%.
295. Comparative strength calculations, and researches on exactness of analyses: theoretical, numerical and experimental. Recommendations of application better theories and calculations methods. 100%.

List of references of the other authors

Attention: wider lists of eferences can be found in above author's papers.

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